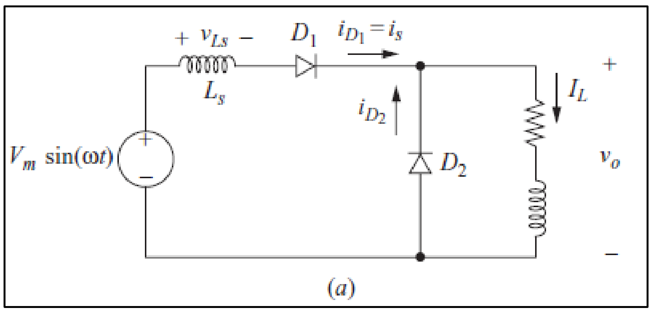
Lecture -4-

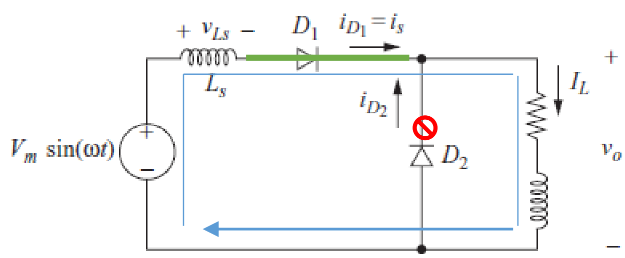
6) The Effect of Source Inductance (Commutation)

The preceding discussion on half-wave rectifiers assumed an ideal source. In practical circuits, the source has an equivalent impedance which is mostly inductive reactance. the non-ideal circuit is analyzed by including the source inductance with the load elements. However, the source inductance Ls causes a fundamental change in circuit behavior for circuits like the half-wave rectifier with a freewheeling diode as shown in Fig.a.

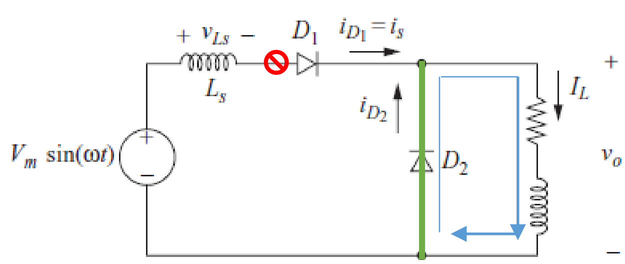


(a)

- As explained previously in the analyzing of the free-wheeling diode, at Vs < 0 (forward biased), the source current flows through the circuit during D_1 to the load.



- at Vs < 0 (reverse biased), D_1 is off and D_2 is on, the load current is I_L

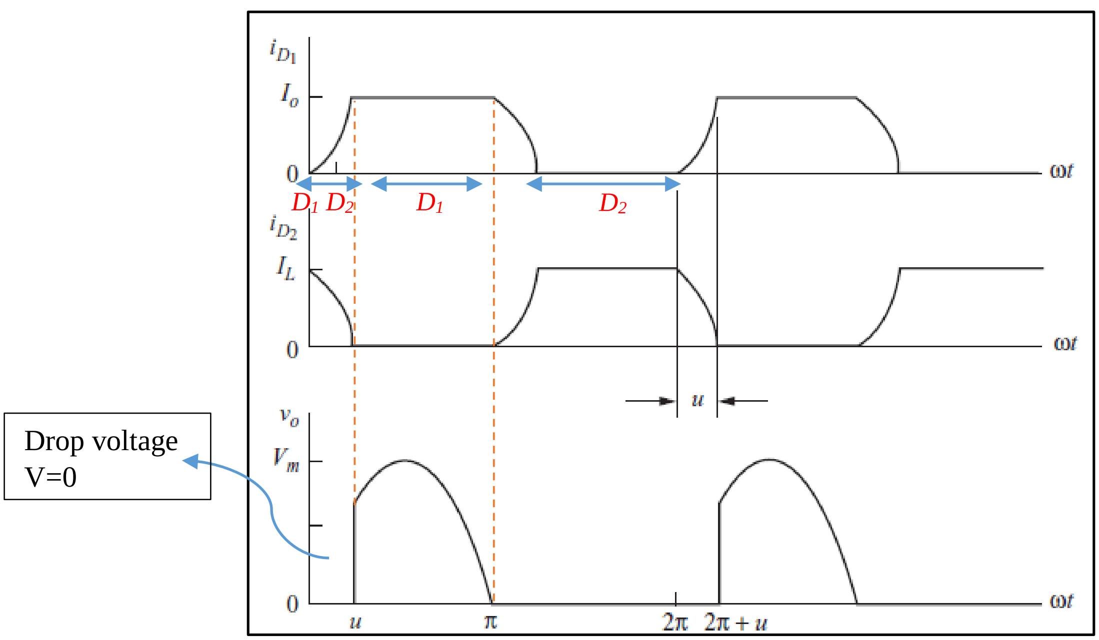


- When the diode turns off (as the AC voltage drops to zero), if there is significant source inductance, the current may not immediately drop to zero due to the energy stored in the inductance. In this instant, the other diode has been turned on which allows the current to flow through. This creates a situation (a slow drop of inductance current with a current flowing at the other diode conduction), leading to a phenomenon known as "current overlap." The interval when both D_1 and D_2 are on is called the commutation time or **commutation angle (u).** There will be a sharp rise in the current wave. as shown in Fig.b.

- "Current overlap." can refer to the time that the current through the inductance is still flowing even after the diode has turned off.

- **Commutation in a rectifier** is the process of transfer of current from one electronic device (diode or thyristor) to the another. Commutation, in this case, is an example of natural commutation or line commutation, where the change in instantaneous line voltage results in a device turning off naturally. This is common in AC circuits where the alternating current periodically reverses direction.

- Disadvantages of this circuit is waveform Distortion: The presence of inductance can lead to a non-sinusoidal current waveform. This may result in harmonic distortion, which can affect other equipment connected to the same supply.

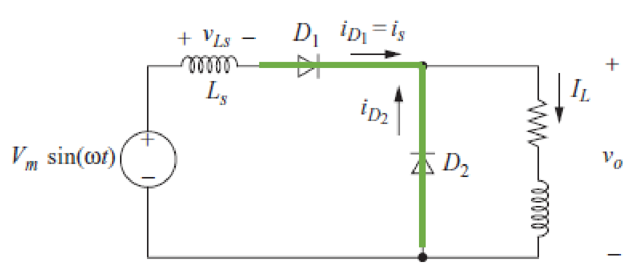


b) Diode currents and load voltage showing the effects of Commutation.

When both D_1 and D_2 are on, the voltage across Ls is

$$v_{Ls} = V_m \sin(\omega t)$$

$$wL\frac{di_s(wt)}{dt} = Vm \sin wt$$

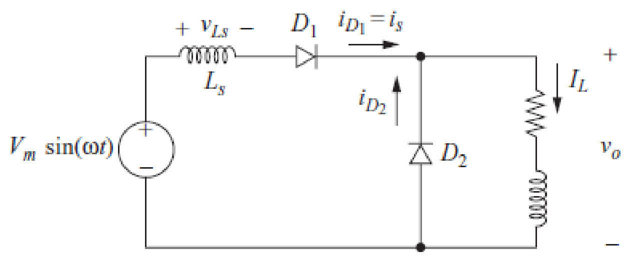


$$\frac{di_s(wt)}{dt} = \frac{Vm}{wL}\ sin\, wt$$

$$\int \frac{di_s(wt)}{dt} = \int \frac{Vm}{wL}\ sin\, wt\ .d(wt)$$

$$i_s(wt) = \frac{Vm}{wL}\int_0^{wt} sin\, wt\ .d(wt) = -\frac{Vm}{wL}\ [\cos wt - 1]$$

$$i_s(wt) = \frac{Vm}{wL}\ [1 - \cos wt]$$



Current in D_2 is

$$i_L = i_{D2} + i_s$$

$$i_{D_2} = I_L - i_s = I_L - \frac{V_m}{\omega L_s}(1 - \cos \omega t)$$

The current in D_2 starts at I_L and decreases to zero. Letting the angle at which the current reaches zero be $wt = u$ (commutation angle)

$$i_{D_2}(u) = I_L - \frac{V_m}{\omega L_s}(1 - \cos u) = 0$$

$$I_L = \frac{Vm}{wL_s}\ [1 - \cos u]$$

$$[[\ I_L = \frac{Vm}{wL_s} - \frac{Vm}{wL_s}\cos u\ \ ]] * \frac{wL_s}{Vm}$$

$$\frac{wL_s}{Vm} I_L = 1 - \cos u$$

$$\cos u = 1 - \frac{wL_s}{Vm} I_L$$

$$\boxed{u = \cos^{-1}\left(1 - \frac{I_L \omega L_s}{V_m}\right) = \cos^{-1}\left(1 - \frac{I_L X_s}{V_m}\right)}$$

The commutation angle affects the voltage across the load. Since the voltage across the load is zero when D_2 is conducting, the load voltage remains at zero through the commutation angle, as shown in Fig. b. Recall that the load voltage is a half-wave rectified sinusoid when the source is ideal.

تؤثر زاوية التبديل على الجهد عبر الحمل. نظرًا لأن الجهد عبر الحمل يساوي صفرًا عندما يكون D2 موصلًا، فإن جهد الحمل يظل عند الصفر عبر زاوية التبديل، كما هو موضح في الشكل (b). تذكر أن جهد الحمل عبارة عن جيب تمام نصف الموجة عندما يكون المصدر مثاليًا.

Average load voltage is

$$V_o = \frac{1}{2\pi}\int_u^{\pi} V_m \sin(\omega t)\, d(\omega t)$$

$$= \frac{V_m}{2\pi}[-\cos(\omega t)]\Big|_u^{\pi} = \frac{V_m}{2\pi}(1 + \cos u)$$

Using *u* Eq.

$$Vo = \frac{Vm}{2\pi}\left(1 + 1 - \frac{wL_s}{Vm} I_L\right) = \frac{Vm}{2\pi}\left(2 - \frac{I_L X_s}{Vm}\right)$$

$$\boxed{V_o = \frac{V_m}{\pi}\left(1 - \frac{I_L X_s}{2V_m}\right)}$$

Recall that the average of a half-wave rectified sine wave is Vm/π . Source reactance thus reduces average load voltage.

Example:

The half-wave rectifier with freewheeling diode has a 120 V rms ac source that has an inductance of 1.5 mH at 60 Hz. The load current is a constant 5 A. Determine the commutation angle and the average output voltage.

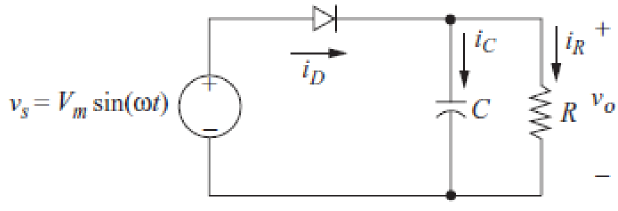
$$u = \cos^{-1}\left(1 - \frac{I_L X_s}{V_m}\right); \qquad X_s = \omega L_s = 377(1.5)(10)^{-3} = 0.566\ \Omega$$

$$u = \cos^{-1}\left(1 - \frac{5(0.452)}{120\sqrt{2}}\right) = 10.47^{\circ}$$

$$V_o = \frac{V_m}{\pi}\left(1 - \frac{X_L X_s}{2V_m}\right) = \frac{120\sqrt{2}}{\pi}\left(1 - \frac{5(.566)}{2\sqrt{2}(120)}\right) = 53.57\ V.$$

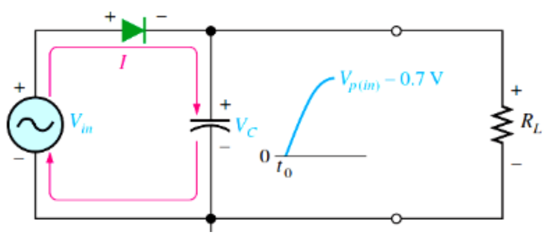
7) *Half-wave Rectifier with a Capacitor Filter (RC- load).*

- The filtering circuit consists of an Ac source, a diode, a resistor, and a capacitor as shown in figure below (a). The resistance may represent an external load, and the capacitor may be a filter which is part of the rectifier circuit.

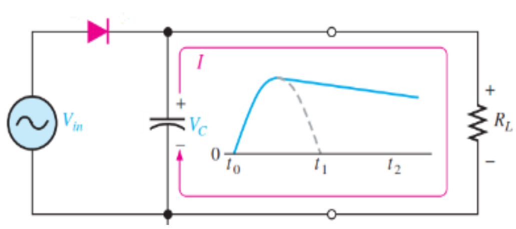


(a)

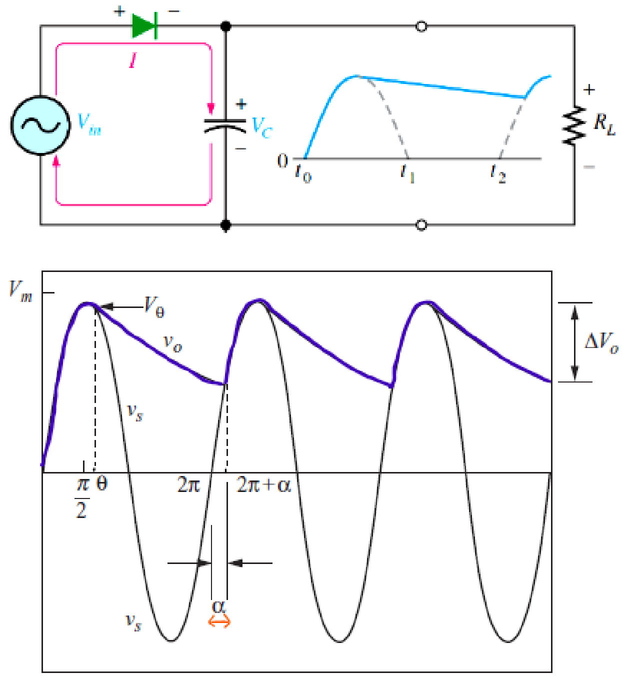
- The output voltage of the half-wave Rectifier circuit is a DC voltage, but it is not completely pure DC, so the ripples observed in the output need to be eliminated.
- The purpose of using capacitor is to reduce the variation (filtering) in the output voltage, making it more like dc. As C increases, the conduction interval for the diode decreases. Therefore, increasing the capacitance to reduce the output voltage ripple results in a larger peak diode current.
- Assuming the capacitor is initially uncharged and the circuit is energized at ωt=0, the diode becomes forward-biased as the source becomes positive. With the diode on, the capacitor charges and the output voltage is the same as the source voltage. The capacitor is charged to Vm when the input voltage reaches its positive peak at ωt=π/2 as shown in figure below.



- As the source decreases after ωt=π/2, the capacitor discharges into the load resistor. At some point, the voltage of the source becomes less than the output voltage, leading to reverse-biasing the diode and isolating the load from the source. The output voltage shown in figure below is a decaying exponential with time constant RC while the diode is off.



- (c) The capacitor charges back to peak of input when the diode becomes forward-biased. This charging occurs during the portion of the input voltage indicated by the solid dark blue curve.



(b)

Figure (*a*) Half-wave rectifier with *RC* load; (*b*) Input and output voltages.

The angle $\omega t = \theta$ is the point when the diode turns off. The output voltage is described by

where $$V_\theta = V_m \sin\theta$$

$$v_o(\omega t) = \begin{cases} V_m \sin \omega t & \text{diode on} \quad \textbf{\textit{(charging)}} \\ V_\theta e^{-(\omega t-\theta)/\omega RC} & \text{diode off} \quad \textbf{\textit{(discharging)}} \end{cases}$$

$(\omega t - \theta) \rightarrow\rightarrow$ due to not starting from 0.

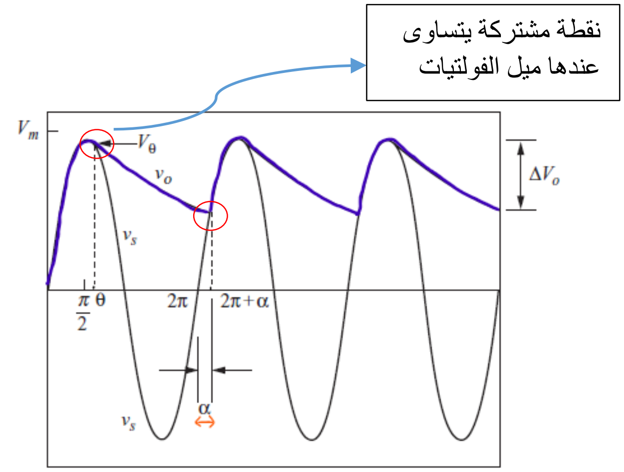
<u>**To find angle θ**</u>:

θ is conduction angle between charging and discharging of the capacitor, so The slopes of these functions are (derivative of functions):

$$\frac{d}{d(\omega t)}[V_m \sin(\omega t)] = V_m \cos(\omega t) \qquad \textbf{\textit{(charging)}}$$

$$\frac{d}{d(\omega t)}\left(V_m \sin\theta\, e^{-(\omega t-\theta)/\omega RC}\right) = V_m \sin\theta\left(-\frac{1}{\omega RC}\right)e^{-(\omega t-\theta)/\omega RC} \qquad \textbf{\textit{(discharging)}}$$

At [$\omega t = \theta$], the decreasing rate of source voltage and capacitor voltage are equal, So the slopes of these voltage functions are:



(b)

$$V_m \cos\theta = \left(\frac{V_m \sin\theta}{-\omega RC}\right) e^{-(\theta-\theta)/\omega RC} = \frac{V_m \sin\theta}{-\omega RC}$$

$$\frac{V_m \cos\theta}{V_m \sin\theta} = \frac{1}{-\omega RC}$$

$$\frac{1}{\tan\theta} = \frac{1}{-\omega RC}$$

$$\boxed{\theta = \tan^{-1}(-\omega RC) = -\tan^{-1}(\omega RC) + \pi}$$

In practical circuits where the time constant is large,

$$\boxed{\theta \approx \frac{\pi}{2} \quad \text{and} \quad V_m \sin\theta \approx V_m}$$

When the source voltage comes back up to the value of the output voltage in the next period, the diode becomes forward-biased, and the output again is the same as the source voltage. The angle at which the diode turns on in the second period, [$\omega t = 2\pi + \alpha$].

The current in the resistor is calculated from *iR =Vo*R*. The current in the capacitor is calculated from

$$i_C(t) = C\frac{dv_o(t)}{dt}$$

which can also be expressed, using ωt as the variable, as

$$i_C(\omega t) = \omega C\frac{dv_o(\omega t)}{d(\omega t)}$$

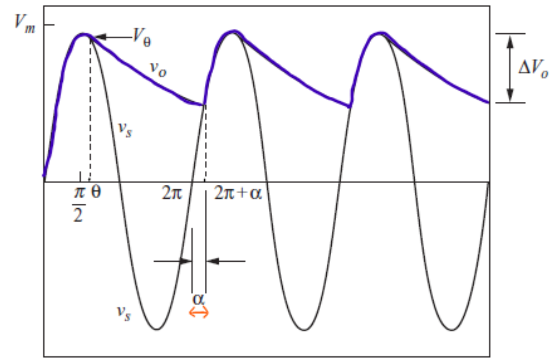
Using *Vo*

$$i_C(\omega t) = \begin{cases} -\left(\frac{V_m \sin\theta}{R}\right) e^{-(\omega t-\theta)/\omega RC} & \text{for } \theta \le \omega t \le 2\pi + \alpha \quad \text{(diode off)} \\ \omega C V_m \cos(\omega t) & \text{for } 2\pi + \alpha \le \omega t \le 2\pi + \theta \quad \text{(diode on)} \end{cases}$$ ………….(#)

The source current, which is the same as the diode current, is

$$i_S = i_D = i_R + i_C$$

Peak capacitor current occurs when the diode **turns on** at [ωt = 2π + α].From equation (#)



(b)

$$I_{C,\text{peak}} = \omega C V_m \cos(2\pi + \alpha) = \omega C V_m \cos\alpha$$

Resistor current at [ωt = 2π + α]. is obtained

$$iR(2\pi + \alpha) = \frac{Vm\sin(2\pi + \alpha)}{R} = \frac{Vm\sin\alpha}{R}$$

Peak diode current is

$$I_{D,\text{ peak}} = \omega C V_m \cos\alpha + \frac{V_m \sin\alpha}{R} = V_m\left(\omega C\cos\alpha + \frac{\sin\alpha}{R}\right)$$

The effectiveness of the capacitor filter is determined by the variation in output voltage. This may be expressed as the difference between the maximum and minimum output voltage, which is the peak-to-peak ripple voltage.

For the half wave rectifier ,the maximum output voltage is ***Vm***. The minimum output voltage occurs at [**ωt = 2π + α],** which can be computed from ***Vm sin α***. The peak-to-peak ripple for the circuit of Fig. a is expressed as:

$$\Delta V_o = V_m - V_m\sin\alpha = V_m(1 - \sin\alpha)$$

In circuits where the capacitor is selected to provide for a nearly constant dc output voltage, the RC time constant is large compared to the period of the sine wave, Moreover, the diode turns on close to the peak of the sine wave when $\alpha \approx \pi/2$. The change in output voltage when the diode is off, if $V_\theta \approx V_m$ and $\theta \approx \pi/2$, $\alpha = \pi/2$

$$v_o(\omega t) = \begin{cases} V_m \sin\omega t & \text{diode on} \\ V_\theta e^{-(\omega t - \theta)/\omega RC} & \text{diode off} \end{cases}$$

where $$V_\theta = V_m \sin\theta$$

$$v_o(2\pi + \alpha) = V_m e^{-(2\pi + \pi/2 - \pi/2)/\omega RC} = V_m e^{-2\pi/\omega RC}$$

The ripple voltage can then be approximated as

$$\Delta V_o \approx V_m - V_m e^{-2\pi/\omega RC} = V_m\left(1 - e^{-2\pi/\omega RC}\right)$$

Furthermore, the exponential in the above equation can be approximated by the series expansion:

$$e^{-2\pi/\omega RC} \approx 1 - \frac{2\pi}{\omega RC}$$

Substituting for the exponential in Eq, the peak-to-peak ripple is approximately

$$\boxed{\Delta V_o \approx V_m\left(\frac{2\pi}{\omega RC}\right) = \frac{V_m}{fRC}}$$

w=2πf

The output voltage ripple is reduced by increasing the filter capacitor C. As C increases, the conduction interval for the diode decreases. Therefore, increasing the capacitance to reduce the output voltage ripple results in a larger peak diode current.

Example: The half-wave rectifier with a capacitor filter has a 120-V rms source at 60 Hz, R=500 Ω, and C=100μF. Determine (a) an expression for output voltage, (b) the peak-to-peak voltage variation on the output, (c) an expression for capacitor current, (d) the peak diode current, and (e) the value of C such that ΔVo is 1 percent of Vm. $\alpha = 0.843 \text{ rad} = 48°$

Solution/

(*a*) Output voltage is expressed from Eq

$$v_o(\omega t) = \begin{cases} V_m \sin \omega t & \text{diode on} \\ V_\theta e^{-(\omega t - \theta)/\omega RC} & \text{diode off} \end{cases}$$

$$V_m = 120\sqrt{2} = 169.7 \text{ V}$$

$$\omega RC = (2\pi 60)(500)(10)^{-6} = 18.85 \text{ rad}$$ *100

$$\boxed{\theta = \tan^{-1}(-\omega RC) = -\tan^{-1}(\omega RC) + \pi}$$

$$\theta = -\tan^{-1}(18.85) + \pi = 1.62 \text{ rad} = 93°$$

$$V_m \sin\theta = 169.5 \text{ V}$$

$$v_o(\omega t) = \begin{cases} 169.7 \sin(\omega t) & 2\pi + \alpha \le \omega t \le 2\pi + \theta \\ 169.5 e^{-(\omega t - 1.62)/18.85} & \theta \le \omega t \le 2\pi + \alpha \end{cases}$$

(charging)

(discharging)

(b) Peak-to-peak output voltage is described by Eq.

$$\Delta V_o = V_m(1 - \sin\alpha) = 169.7(1 - \sin 0.843) = 43 \text{ V}$$

(c) The capacitor current is determined from Eq.

$$i_C(\omega t) = \begin{cases} -\left(\dfrac{V_m \sin\theta}{R}\right)e^{-(\omega t-\theta)/\omega RC} & \text{for } \theta \le \omega t \le 2\pi + \alpha \quad \text{(diode off)} \\ \omega C V_m \cos(\omega t) & \text{for } 2\pi + \alpha \le \omega t \le 2\pi + \theta \quad \text{(diode on)} \end{cases}$$

$$i_C(\omega t) = \begin{cases} -0.339e^{-(\omega t-1.62)/18.85} \quad \text{A} & \theta \le \omega t \le 2\pi + \alpha \\ 6.4\cos(\omega t) \quad \text{A} & 2\pi + \alpha \le \omega t \le 2\pi + \theta \end{cases}$$

(d) Peak diode current is determined from Eq.

$$I_{D,\text{peak}} = \omega C V_m \cos\alpha + \frac{V_m \sin\alpha}{R} = V_m\left(\omega C \cos\alpha + \frac{\sin\alpha}{R}\right)$$

$$I_{D,\text{peak}} = \sqrt{2}(120)\left[377(10)^{-4}\cos 0.843 + \frac{\sin 8.43}{500}\right]$$

$$= 4.26 + 0.34 = 4.50 \text{ A}$$

(e) the value of C such that ΔVo is 1 percent of Vm.

$$\Delta V_o \approx V_m\left(\frac{2\pi}{\omega RC}\right) = \frac{V_m}{fRC}$$

$$C \approx \frac{V_m}{fR(\Delta V_o)} = \frac{V_m}{(60)(500)(0.01V_m)} = \frac{1}{300}\text{F} = 3333\ \mu\text{F}$$

Summary

- A rectifier converts ac to dc. Power transfer is from the ac source to the dc load.
- The half-wave rectifier with a resistive load has an average load voltage of V_m/π and an average load current of $V_m/\pi R$.
- The current in a half-wave rectifier with an RL load contains a natural and a forced response, resulting in

$$i(\omega t) = \begin{cases} \dfrac{V_m}{Z}\left[\sin(\omega t - \theta) + \sin(\theta)e^{-\omega t/\omega\tau}\right] & \text{for } 0 \leq \omega t \leq \beta \\ 0 & \text{for } \beta \leq \omega t \leq 2\pi \end{cases}$$

where $\quad Z = \sqrt{R^2 + (\omega L)^2}, \qquad \theta = \tan^{-1}\left(\dfrac{\omega L}{R}\right) \qquad$ and $\qquad \tau = \dfrac{L}{R}$

The diode remains on as long as the current is positive. Power in the RL load is $I_{rms}^2 R$.

- A half-wave rectifier with an RL-source load does not begin to conduct until the ac source reaches the dc voltage in the load. Power in the resistance is $I_{rms}^2 R$, and power absorbed by the dc source is $I_o V_{dc}$, where I_o is the average load current. The load current is expressed as

$$i(\omega t) = \begin{cases} \dfrac{V_m}{Z}\sin(\omega t - \theta) - \dfrac{V_{dc}}{R} + Ae^{-\omega t/\omega\tau} & \text{for } \alpha \leq \omega t \leq \beta \\ 0 & \text{otherwise} \end{cases}$$

where

$$A = \left[-\frac{V_m}{Z}\sin(\alpha - \beta) + \frac{V_{dc}}{R}\right]e^{\alpha/\omega\tau}$$

- A freewheeling diode forces the voltage across an RL load to be a half-wave rectified sine wave. The load current can be analyzed using Fourier analysis. A large load inductance results in a nearly constant load current.
- A large filter capacitor across a resistive load makes the load voltage nearly constant. Average diode current must be the same as average load current, making the peak diode current large.