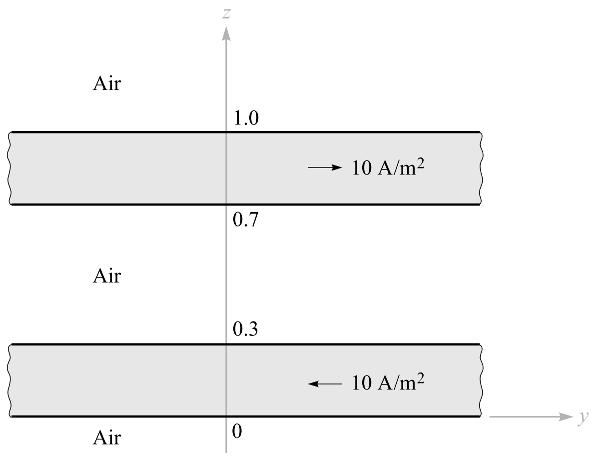

Figure 7.22 See Problem 7.12.

$\rho = 1\,\text{cm}$, $-250\,\text{mA/m}$ at $\rho = 2\,\text{cm}$, and $-300\,\text{mA/m}$ at $\rho = 3\,\text{cm}$. Calculate H_ϕ at $\rho = 0.5$, 1.5, 2.5, and 3.5 cm.

7.12 In Figure 7.22, let the regions $0 < z < 0.3$ m and $0.7 < z < 1.0$ m be conducting slabs carrying uniform current densities of 10 A/m^2 in opposite directions as shown. Find **H** at $z =$: (*a*) -0.2; (*b*) 0.2; (*c*) 0.4; (*d*) 0.75; (*e*) 1.2 m.

7.13 A hollow cylindrical shell of radius a is centered on the z axis and carries a uniform surface current density of $K_a\mathbf{a}_\phi$. (*a*) Show that H is not a function of ϕ or z. (*b*) Show that H_ϕ and H_ρ are everywhere zero. (*c*) Show that $H_z = 0$ for $\rho > a$. (*d*) Show that $H_z = K_a$ for $\rho < a$. (*e*) A second shell, $\rho = b$, carries a current $K_b\mathbf{a}_\phi$. Find **H** everywhere.

7.14 A toroid having a cross section of rectangular shape is defined by the following surfaces: the cylinders $\rho = 2$ and $\rho = 3$ cm, and the planes $z = 1$ and $z = 2.5$ cm. The toroid carries a surface current density of $-50\mathbf{a}_z$ A/m on the surface $\rho = 3$ cm. Find **H** at the point $P(\rho, \phi, z)$: (*a*) $P_A(1.5\,\text{cm}, 0, 2\,\text{cm})$; (*b*) $P_B(2.1\,\text{cm}, 0, 2\,\text{cm})$; (*c*) $P_C(2.7\,\text{cm}, \pi/2, 2\,\text{cm})$; (*d*) $P_D(3.5\,\text{cm}, \pi/2, 2\,\text{cm})$.

7.15 Assume that there is a region with cylindrical symmetry in which the conductivity is given by $\sigma = 1.5e^{-150\rho}$kS/m. An electric field of $30\mathbf{a}_z$ V/m is present. (*a*) Find **J**. (*b*) Find the total current crossing the surface $\rho < \rho_0$, $z = 0$, all ϕ. (*c*) Make use of Ampère's circuital law to find **H**.

7.16 A current filament carrying I in the $-\mathbf{a}_z$ direction lies along the entire positive z axis. At the origin, it connects to a conducting sheet that forms the xy plane. (*a*) Find **K** in the conducting sheet. (*b*) Use Ampere's circuital law to find **H** everywhere for $z > 0$; (*c*) find **H** for $z < 0$.

7.17 A current filament on the z axis carries a current of 7 mA in the $\mathbf{a}_z$ direction, and current sheets of 0.5 $\mathbf{a}_z$ A/m and -0.2 $\mathbf{a}_z$ A/m are located at $\rho = 1$ cm and $\rho = 0.5$ cm, respectively. Calculate $\mathbf{H}$ at: (a) $\rho = 0.5$ cm; (b) $\rho = 1.5$ cm; (c) $\rho = 4$ cm. (d) What current sheet should be located at $\rho = 4$ cm so that $\mathbf{H} = 0$ for all $\rho > 4$ cm?

7.18 A wire of 3 mm radius is made up of an inner material ($0 < \rho < 2$ mm) for which $\sigma = 10^7$ S/m, and an outer material (2 mm $< \rho <$ 3 mm) for which $\sigma = 4 \times 10^7$ S/m. If the wire carries a total current of 100 mA dc, determine $\mathbf{H}$ everywhere as a function of ρ.

7.19 In spherical coordinates, the surface of a solid conducting cone is described by $\theta = \pi/4$ and a conducting plane by $\theta = \pi/2$. Each carries a total current I. The current flows as a surface current radially inward on the plane to the vertex of the cone, and then flows radially outward throughout the cross section of the conical conductor. (a) Express the surface current density as a function of r; (b) express the volume current density inside the cone as a function of r; (c) determine $\mathbf{H}$ as a function of r and θ in the region between the cone and the plane; (d) determine $\mathbf{H}$ as a function of r and θ inside the cone.

7.20 A solid conductor of circular cross section with a radius of 5 mm has a conductivity that varies with radius. The conductor is 20 m long, and there is a potential difference of 0.1 V dc between its two ends. Within the conductor, $\mathbf{H} = 10^5 \rho^2 \mathbf{a}_\phi$ A/m. (a) Find σ as a function of ρ. (b) What is the resistance between the two ends?

7.21 A cylindrical wire of radius a is oriented with the z axis down its center line. The wire carries a nonuniform current down its length of density $\mathbf{J} = b\rho\, \mathbf{a}_z$ A/m^2 where b is a constant. (a) What total current flows in the wire? (b) Find $\mathbf{H}_{in}$ ($0 < \rho < a$), as a function of ρ; (c) find $\mathbf{H}_{out}(\rho > a)$, as a function of ρ; (d) verify your results of parts (b) and (c) by using $\nabla \times \mathbf{H} = \mathbf{J}$.

7.22 A solid cylinder of radius a and length L, where $L \gg a$, contains volume charge of uniform density ρ_0 C/m^3. The cylinder rotates about its axis (the z axis) at angular velocity Ω rad/s. (a) Determine the current density $\mathbf{J}$ as a function of position within the rotating cylinder. (b) Determine $\mathbf{H}$ on-axis by applying the results of Problem 7.6. (c) Determine the magnetic field intensity $\mathbf{H}$ inside and outside. (d) Check your result of part (c) by taking the curl of $\mathbf{H}$.

7.23 Given the field $\mathbf{H} = 20\rho^2 \mathbf{a}_\phi$ A/m: (a) Determine the current density $\mathbf{J}$. (b) Integrate $\mathbf{J}$ over the circular surface $\rho \le 1, 0 < \phi < 2\pi, z = 0$, to determine the total current passing through that surface in the $\mathbf{a}_z$ direction. (c) Find the total current once more, this time by a line integral around the circular path $\rho = 1, 0 < \phi < 2\pi, z = 0$.

7.24 Infinitely long filamentary conductors are located in the $y = 0$ plane at $x = n$ meters where $n = 0, \pm 1, \pm 2, \ldots$ Each carries 1 A in the $\mathbf{a}_z$ direction.

(a) Find $\mathbf{H}$ on the y axis. As a help,

$$\sum_{n=1}^{\infty} \frac{y}{y^2+n^2} = \frac{\pi}{2} - \frac{1}{2y} + \frac{\pi}{e^{2\pi y}-1}$$

(b) Compare your result of part (a) to that obtained if the filaments are replaced by a current sheet in the $y = 0$ plane that carries surface current density $\mathbf{K} = 1\mathbf{a}_z$ A/m.

7.25 When x, y, and z are positive and less than 5, a certain magnetic field intensity may be expressed as $\mathbf{H} = [x^2yz/(y+1)]\mathbf{a}_x + 3x^2z^2\mathbf{a}_y - [xyz^2/(y+1)]\mathbf{a}_z$. Find the total current in the $\mathbf{a}_x$ direction that crosses the strip $x = 2$, $1 \le y \le 4$, $3 \le z \le 4$, by a method utilizing: (a) a surface integral; (b) a closed line integral.

7.26 Consider a sphere of radius $r = 4$ centered at (0, 0, 3). Let S_1 be that portion of the spherical surface that lies above the xy plane. Find $\int_{S_1}(\nabla \times \mathbf{H}) \cdot d\mathbf{S}$ if $\mathbf{H} = 3\rho\,\mathbf{a}_\phi$ in cylindrical coordinates.

7.27 The magnetic field intensity is given in a certain region of space as $\mathbf{H} = [(x+2y)/z^2]\mathbf{a}_y + (2/z)\mathbf{a}_z$ A/m. (a) Find $\nabla \times \mathbf{H}$. (b) Find $\mathbf{J}$. (c) Use $\mathbf{J}$ to find the total current passing through the surface $z = 4$, $1 \le x \le 2$, $3 \le z \le 5$, in the $\mathbf{a}_z$ direction. (d) Show that the same result is obtained using the other side of Stokes' theorem.

7.28 Given $\mathbf{H} = (3r^2/\sin\theta)\mathbf{a}_\theta + 54r\cos\theta\,\mathbf{a}_\phi$ A/m in free space: (a) Find the total current in the $\mathbf{a}_\theta$ direction through the conical surface $\theta = 20^\circ$, $0 \le \phi \le 2\pi$, $0 \le r \le 5$, by whatever side of Stokes' theorem you like the best. (b) Check the result by using the other side of Stokes' theorem.

7.29 A long, straight, nonmagnetic conductor of 0.2 mm radius carries a uniformly distributed current of 2 A dc. (a) Find $\mathbf{J}$ within the conductor. (b) Use Ampère's circuital law to find $\mathbf{H}$ and $\mathbf{B}$ within the conductor. (c) Show that $\nabla \times \mathbf{H} = \mathbf{J}$ within the conductor. (d) Find $\mathbf{H}$ and $\mathbf{B}$ outside the conductor. (e) Show that $\nabla \times \mathbf{H} = \mathbf{J}$ outside the conductor.

7.30 (An inversion of Problem 7.20.) A solid, nonmagnetic conductor of circular cross section has a radius of 2 mm. The conductor is inhomogeneous, with $\sigma = 10^6(1 + 10^6\rho^2)$ S/m. If the conductor is 1 m in length and has a voltage of 1 mV between its ends, find: (a) $\mathbf{H}$ inside; (b) the total magnetic flux inside the conductor.

7.31 The cylindrical shell defined by 1 cm $< \rho <$ 1.4 cm consists of a nonmagnetic conducting material and carries a total current of 50 A in the $\mathbf{a}_z$ direction. Find the total magnetic flux crossing the plane $\phi = 0$, $0 < z < 1$: (a) $0 < \rho <$ 1.2 cm; (b) 1.0 cm $< \rho <$ 1.4 cm; (c) 1.4 cm $< \rho <$ 20 cm.

7.32 The free space region defined by $1 < z < 4$ cm and $2 < \rho < 3$ cm is a toroid of rectangular cross section. Let the surface at $\rho = 3$ cm carry a surface current $\mathbf{K} = 2\mathbf{a}_z$ kA/m. (a) Specify the current densities on the surfaces at

$\rho = 2$ cm, $z = 1$ cm, and $z = 4$ cm. (*b*) Find **H** everywhere. (*c*) Calculate the total flux within the toroid.

7.33 Use an expansion in rectangular coordinates to show that the curl of the gradient of any scalar field G is identically equal to zero.

7.34 A filamentary conductor on the z axis carries a current of 16 A in the $\mathbf{a}_z$ direction, a conducting shell at $\rho = 6$ carries a total current of 12 A in the $-\mathbf{a}_z$ direction, and another shell at $\rho = 10$ carries a total current of 4 A in the $-\mathbf{a}_z$ direction. (*a*) Find **H** for $0 < \rho < 12$. (*b*) Plot H_ϕ versus ρ. (*c*) Find the total flux Φ crossing the surface $1 < \rho < 7$, $0 < z < 1$, at fixed ϕ.

7.35 A current sheet, $\mathbf{K} = 20\,\mathbf{a}_z$ A/m, is located at $\rho = 2$, and a second sheet, $\mathbf{K} = -10\mathbf{a}_z$ A/m, is located at $\rho = 4$. (*a*) Let $V_m = 0$ at $P(\rho = 3, \phi = 0, z = 5)$ and place a barrier at $\phi = \pi$. Find $V_m(\rho, \phi, z)$ for $-\pi < \phi < \pi$. (*b*) Let $\mathbf{A} = 0$ at P and find $\mathbf{A}(\rho, \phi, z)$ for $2 < \rho < 4$.

7.36 Let $\mathbf{A} = (3y - z)\mathbf{a}_x + 2xz\mathbf{a}_y$ Wb/m in a certain region of free space. (*a*) Show that $\nabla \cdot \mathbf{A} = 0$. (*b*) At $P(2, -1, 3)$, find **A**, **B**, **H**, and **J**.

7.37 Let $N = 1000$, $I = 0.8$ A, $\rho_0 = 2$ cm, and $a = 0.8$ cm for the toroid shown in Figure 7.12*b*. Find V_m in the interior of the toroid if $V_m = 0$ at $\rho = 2.5$ cm, $\phi = 0.3\pi$. Keep ϕ within the range $0 < \phi < 2\pi$.

7.38 A square filamentary differential current loop, dL on a side, is centered at the origin in the $z = 0$ plane in free space. The current I flows generally in the $\mathbf{a}_\phi$ direction. (*a*) Assuming that $r >> dL$, and following a method similar to that in Section 4.7, show that

$$d\mathbf{A} = \frac{\mu_0 I (dL)^2 \sin\theta}{4\pi r^2}\,\mathbf{a}_\phi$$

(*b*) Show that

$$d\mathbf{H} = \frac{I(dL)^2}{4\pi r^3}(2\cos\theta\,\mathbf{a}_r + \sin\theta\,\mathbf{a}_\theta)$$

The square loop is one form of a *magnetic dipole*.

7.39 Planar current sheets of $\mathbf{K} = 30\mathbf{a}_z$ A/m and $-30\mathbf{a}_z$ A/m are located in free space at $x = 0.2$ and $x = -0.2$, respectively. For the region $-0.2 < x < 0.2$ (*a*) find **H**; (*b*) obtain an expression for V_m if $V_m = 0$ at $P(0.1, 0.2, 0.3)$; (*c*) find **B**; (*d*) obtain an expression for **A** if $\mathbf{A} = 0$ at P.

7.40 Show that the line integral of the vector potential **A** about any closed path is equal to the magnetic flux enclosed by the path, or $\oint \mathbf{A} \cdot d\mathbf{L} = \int \mathbf{B} \cdot d\mathbf{S}$.

7.41 Assume that $\mathbf{A} = 50\rho^2\mathbf{a}_z$ Wb/m in a certain region of free space. (*a*) Find **H** and **B**. (*b*) Find **J**. (*c*) Use **J** to find the total current crossing the surface $0 \le \rho \le 1$, $0 \le \phi < 2\pi$, $z = 0$. (*d*) Use the value of H_ϕ at $\rho = 1$ to calculate $\oint \mathbf{H} \cdot d\mathbf{L}$ for $\rho = 1$, $z = 0$.

7.42 Show that $\nabla_2(1/R_{12}) = -\nabla_1(1/R_{12}) = \mathbf{R}_{21}/R_{12}^3$.

7.43 Compute the vector magnetic potential within the outer conductor for the coaxial line whose vector magnetic potential is shown in Figure 7.20 if the outer radius of the outer conductor is $7a$. Select the proper zero reference and sketch the results on the figure.

7.44 By expanding Eq. (58), Section 7.7 in rectangular coordinates, show that (59) is correct.