Fault Analysis

A fault in an electrical power system is the unintentional and undesirable creation of a conducting path (a short circuit) or a blockage of current (an open circuit). The short-circuit fault is typically the most common and is usually implied when most people use the term fault. We restrict our comments to the short-circuit fault.

The causes of faults include lightning, wind damage, trees falling across lines, vehicles colliding with towers or poles, birds shorting out lines, aircraft colliding with lines, vandalism, small animals entering switchgear, and line breaks due to excessive ice loading.

It is important to determine the values of system voltages and currents during faulted conditions so that protective devices may be set to detect and minimize their harmful effects. The time constants of the associated transients are such that sinusoidal steady-state methods may still be used. The method of symmetrical components is particularly suited to fault analysis.

Analysis types

- Power flow - evaluate normal operating conditions.
- Fault analysis - evaluate abnormal operating conditions.

Fault types: there are number of faults that can be happening in the power system network and can be divided as below:

- Balanced faults — Percentage of total faults
  - Three-phase — <5%
- Unbalanced faults
  - Single-line to ground — 60-75%
  - Double-line to ground — 15-25%
  - Line-to-line faults — 5-15%

Results used for:

- Specifying ratings for circuit breakers and fuses protective relay settings.
- Specifying the impedance of transformers and generators.

Magnitude of fault currents depends on:

- The impedance of the network.
- The internal impedances of the generators.
- The resistance of the fault (arc resistance).

Unbalance fault analysis requires new tools

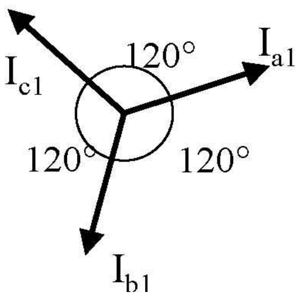
- Symmetrical components
- Augmented component models.

Symmetrical Components

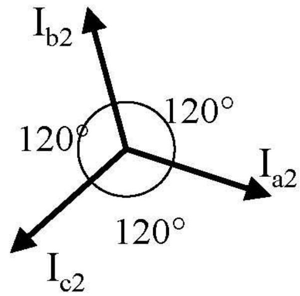
Allow unbalanced three-phase phase quantities to be replaced by the sum of three separate but balanced symmetrical components

- applicable to current and voltages
- permits modeling of unbalanced systems and net works

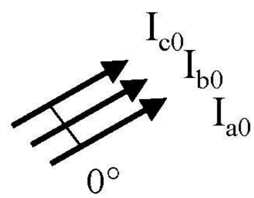
Representative symmetrical components



abc sequence
positive sequence



acb sequence
negative sequence



zero sequence

- **Positive sequence phasors**

$$I_{a1} = |I_{a1}|\angle(\delta + 0^\circ) = I_{a1}$$
$$I_{b1} = |I_{a1}|\angle(\delta + 240^\circ) = a^2 I_{a1}$$
$$I_{c1} = |I_{a1}|\angle(\delta + 120^\circ) = aI_{a1}$$

Operator ***a*** identities

$$a = 1\angle 120^\circ = -0.5 + j0.866$$
$$a^2 = 1\angle 240^\circ = -0.5 - j0.866$$
$$a^3 = 1\angle 0^\circ = 1 + j0$$
$$a + a^2 + a^3 = 0$$

- **Negative sequence phasors**

$$I_{a2} = |I_{a2}|\angle(\delta + 0^\circ) = I_{a2}$$
$$I_{b2} = |I_{a2}|\angle(\delta + 120^\circ) = aI_{a2}$$
$$I_{c2} = |I_{a2}|\angle(\delta + 240^\circ) = a^2 I_{a2}$$

- **Zero sequence phasors**

$$I_{a0} = |I_{a0}|\angle(\delta + 0^\circ) = I_{a0}$$
$$I_{b0} = |I_{a0}|\angle(\delta + 0^\circ) = I_{a0}$$
$$I_{c0} = |I_{a0}|\angle(\delta + 0^\circ) = I_{a0}$$

Relating unbalanced phasors to symmetrical components

$$I_a = I_{a0} + I_{a1} + I_{a2} = I_{a0} + I_{a1} + I_{a2}$$
$$I_b = I_{b0} + I_{b1} + I_{b2} = I_{a0} + a^2 I_{a1} + aI_{a2}$$
$$I_c = I_{c0} + I_{c1} + I_{c2} = I_{a0} + aI_{a1} + a^2 I_{a2}$$

In matrix notation

$$\begin{bmatrix} I_a \\ I_b \\ I_c \end{bmatrix} = \begin{bmatrix} 1 & 1 & 1 \\ 1 & a^2 & a \\ 1 & a & a^2 \end{bmatrix} \begin{bmatrix} I_{a0} \\ I_{a1} \\ L_{a2} \end{bmatrix}$$

[A] is known as the symmetrical components transformation matrix

I_{abc}=A I_{012} A= $\begin{bmatrix} 1 & 1 & 1 \\ 1 & a^2 & a \\ 1 & a & a^2 \end{bmatrix}$

Solving for the symmetrical components leads to

$I_{012}=A^{-1}I_{abc}$

$$A^{-1}=\frac{1}{3}\begin{bmatrix}1 & 1 & 1\\ 1 & a & a^2\\ 1 & a^2 & a\end{bmatrix}=\frac{1}{3}A^*$$

In component form, the calculation for symmetrical components are

$$I_{a0}=\frac{1}{3}(I_a+I_b+I_c)$$

$$I_{a1}=\frac{1}{3}(I_a+aI_b+a^2I_c)$$

$$I_{a2}=\frac{1}{3}(I_a+a^2I_b+aI_c)$$

Similar expressions exist for voltages

$$V_{abc}=AV_{012}$$

$$V_{012}=A^{-1}V_{abc}$$

The apparent power may also be expressed in terms of symmetrical components

$$S_{3\Phi}=V_{abc}^T I_{abc}^*$$

$$S_{3\Phi}=(AV_{012})^T(AI_{012})^*$$

$$S_{3\Phi}=V_{012}^T A^T A^* I_{012}^*$$

Since $A^TA^*=3$

$$S_{3\Phi}=3V_{012}^T I_{012}^*=3V_{a0}I_{a0}^*+3V_{a1}I_{a1}^*+3V_{a2}I_{a2}^*$$

Example 1

Obtain the symmetrical components of a set of unbalanced currents

$$I_a = 1.6\angle 25^\circ$$
$$I_b = 1.0\angle 180^\circ$$
$$I_c = 0.9\angle 132^\circ$$

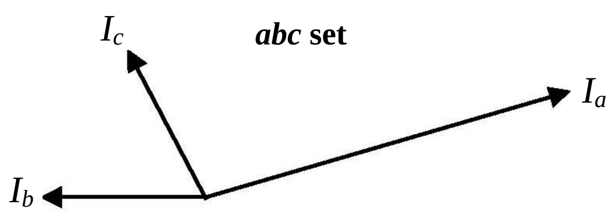
Solution:

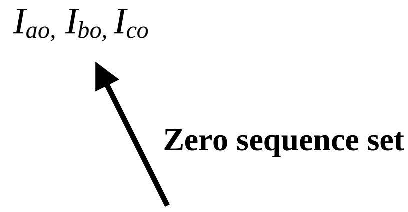
$$I_{012} = \frac{1}{3}\begin{bmatrix} 1 & 1 & 1 \\ 1 & a & a^2 \\ 1 & a^2 & a \end{bmatrix}\begin{bmatrix} I_a \\ I_b \\ I_c \end{bmatrix}$$

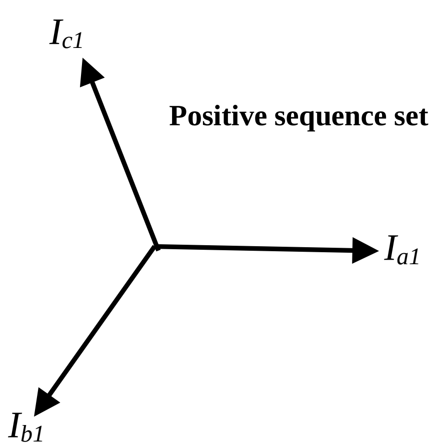
$$I_{a0} = \frac{(1.6\angle 25^\circ) + (1.0\angle 180^\circ) + (0.9\angle 132^\circ)}{3} = 0.45\angle 96.5^\circ$$

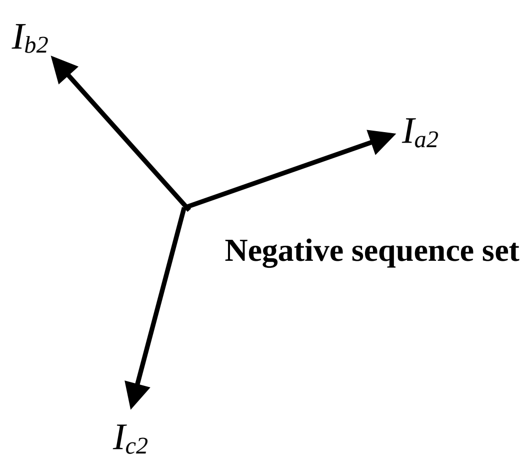
$$I_{a1} = \frac{(1.6\angle 25^\circ) + a(1.0\angle 180^\circ) + a^2(0.9\angle 132^\circ)}{3} = 0.94\angle -0.1^\circ$$

$$I_{a2} = \frac{(1.6\angle 25^\circ) + a^2(1.0\angle 180^\circ) + a(0.9\angle 132^\circ)}{3} = 0.60\angle 22.3^\circ$$









<u>Example 2</u>

The symmetrical components of a set of unbalanced voltages are

$V_{a0} = 0.6\angle 90^\circ$

$V_{a1} = 1.0\angle 30^\circ$

$V_{a2} = 0.8\angle -30^\circ$

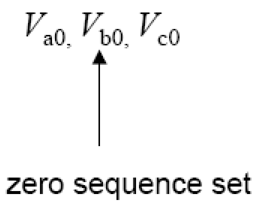
Obtain the original unbalanced voltages:

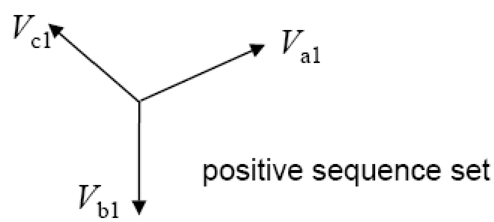
<u>Solution</u>:

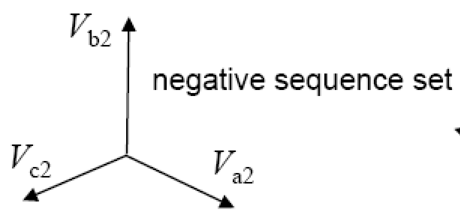
$V_a = (0.6\angle 90^\circ) + (1.0\angle 30^\circ) + (0.8\angle -30^\circ) = 1.7088\angle 24.2^\circ$

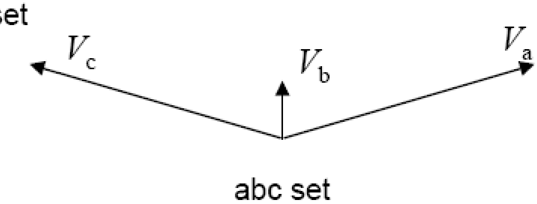
$V_b = (0.6\angle 90^\circ) + a^2(1.0\angle 30^\circ) + a(0.8\angle -30^\circ) = 0.4\angle 90^\circ$

$V_c = (0.6\angle 90^\circ) + a(1.0\angle 30^\circ) + a^2(0.8\angle -30^\circ) = 1.7088\angle 155.8^\circ$









<u>Example 3</u>

Obtain the symmetrical components for the set of unbalanced voltages

$V_a = 300\angle -120^\circ$

$V_b = 200\angle 90^\circ$

$V_c = 100\angle -30^\circ$

Ans.

$V_{a0} = 42.2650\angle -120^\circ$

$V_{a1} = 193.1852\angle -135^\circ$

$V_{a2} = 86.9473\angle -84.8961^\circ$

Example 4

The symmetrical components of a set of unbalanced three-phase currents are:

$I_{a0} = 3\angle -30^\circ$

$I_{a1} = 5\angle 90^\circ$

$I_{a2} = 4\angle 30^\circ$

Obtain the original unbalanced phasors.

Ans.

$I_a = 8.1854\angle 42.2163^\circ$

$I_b = 4\angle -30^\circ$

$I_c = 8.18544\angle -102.2163^\circ$

Example 5

The operator **a** is defined as **a** =1∟120º; show that

(a) $\frac{(1+a)}{(1+a^2)} = 1\angle 120^\circ$

(b) $\frac{(1-a)^2}{(1+a)^2} = 3\angle -180^\circ$

Solution:

(a) Since $1 + a + a^2 = 0$, we have

$$\frac{(1+a)}{(1+a^2)} = \frac{1+a}{-a} = -\frac{1}{a} - 1$$
$$= 0.5 + j0.866 - 1 = 1\angle 120^\circ$$

(b)

$$\frac{(1-a)^2}{(1+a)^2} = \frac{(1-a)^2}{(-a^2)^2} = \frac{1-2a+a^2}{a} = \frac{1}{a} - 2 + a =$$
$$= -0.5 - j0.866 - 2 - 0.5 + j0.866 = 3\angle 180^\circ$$

Sequence Impedances

The impedance offered to the flow of a sequence current creating sequence voltages

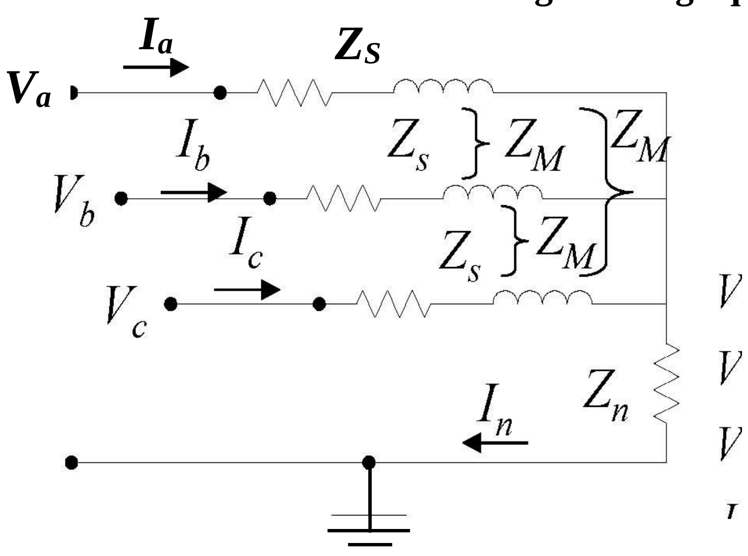
- positive, negative, and zero sequence impedances.

Augmented network models

- wye-connected balanced loads
- transmission line
- 3-phase transformers
- generators

<u>Balanced Loads</u>

Model and governing equations



$$V_a = Z_S I_a + Z_M I_b + Z_M I_c + Z_n I_n$$
$$V_b = Z_M I_a + Z_S I_b + Z_M I_c + Z_n I_n$$
$$V_c = Z_M I_a + Z_M I_b + Z_S I_c + Z_n I_n$$
$$I_n = I_a + I_b + I_c$$

$$\begin{bmatrix} V_a \\ V_b \\ V_c \end{bmatrix} = \begin{bmatrix} Z_S + Z_n & Z_M + Z_n & Z_M + Z_n \\ Z_M + Z_n & Z_S + Z_n & Z_M + Z_n \\ Z_M + Z_n & Z_M + Z_n & Z_S + Z_n \end{bmatrix} \begin{bmatrix} I_a \\ I_b \\ I_c \end{bmatrix}$$

$$V_{abc} = Z_{abc} I_{abc}$$

$$V_{abc} = Z_{abc} I_{abc} \rightarrow (AV_{012}) = Z_{abc}(AI_{012})$$

$$V_{012} = \left[A^{-1} Z_{abc} A\right] I_{012} \rightarrow V_{012} = Z_{012} I_{012}$$

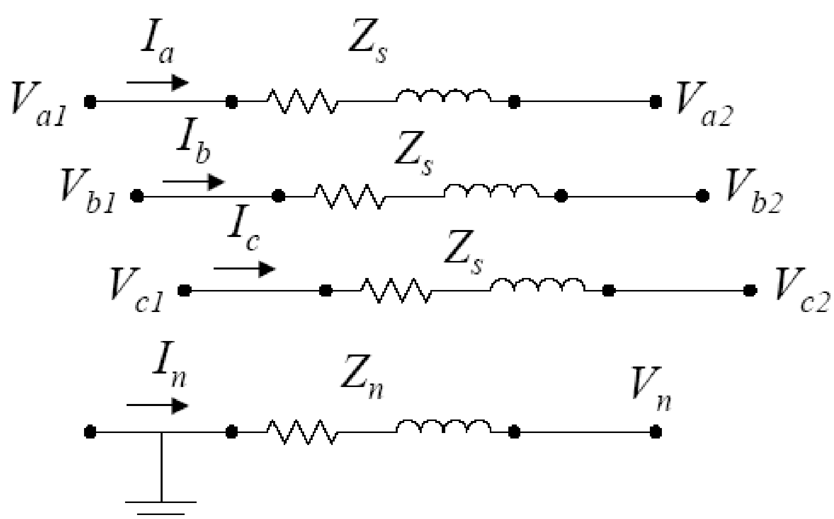
$$Z_{012} = \left[A^{-1} Z_{abc} A\right]$$

$$= \frac{1}{3}\begin{bmatrix} 1 & 1 & 1 \\ 1 & a & a^2 \\ 1 & a^2 & a \end{bmatrix} \begin{bmatrix} Z_S + Z_n & Z_M + Z_n & Z_M + Z_n \\ Z_M + Z_n & Z_S + Z_n & Z_M + Z_n \\ Z_M + Z_n & Z_M + Z_n & Z_S + Z_n \end{bmatrix} \begin{bmatrix} 1 & 1 & 1 \\ 1 & a^2 & a \\ 1 & a & a^2 \end{bmatrix}$$

$$= \begin{bmatrix} Z_S + 3Z_n + 2Z_M & 0 & 0 \\ 0 & Z_S - Z_M & 0 \\ 0 & 0 & Z_S - Z_M \end{bmatrix}$$

<u>Transmission Line</u>

Model and governing equations



$$V_{a1} = Z_S I_a - Z_n I_n + V_{a2}$$
$$V_{b1} = Z_S I_b - Z_n I_n + V_{b2}$$
$$V_{c1} = Z_S I_c - Z_n I_n + V_{c2}$$
$$V_n = 0 + Z_n I_n$$
$$I_n = I_a + I_b + I_c$$

$$\begin{bmatrix} V_{a1} \\ V_{b1} \\ V_{c1} \end{bmatrix} = \begin{bmatrix} Z_S + Z_n & Z_n & Z_n \\ Z_n & Z_S + Z_n & Z_n \\ Z_n & Z_n & Z_S + Z_n \end{bmatrix} \begin{bmatrix} I_a \\ I_b \\ I_c \end{bmatrix} + \begin{bmatrix} V_{a2} \\ V_{b2} \\ V_{c2} \end{bmatrix}$$

$$V_{abc1} = Z_{abc} I_{abc} + V_{abc2}$$
$$V_{abc1} = Z_{abc} I_{abc} + V_{abc2} \rightarrow (AV_{012-1}) = Z_{abc}(AI_{012}) + (AV_{012-2})$$
$$V_{012-1} = A^{-1} Z_{abc} A I_{012} + V_{012-2} = Z_{012} I_{012} + V_{012-2}$$
$$Z_{012} = A^{-1} Z_{abc} A$$
$$= \frac{1}{3} \begin{bmatrix} 1 & 1 & 1 \\ 1 & a & a^2 \\ 1 & a^2 & a \end{bmatrix} \begin{bmatrix} Z_S + Z_n & Z_n & Z_n \\ Z_n & Z_S + Z_n & Z_n \\ Z_n & Z_n & Z_S + Z_n \end{bmatrix} \begin{bmatrix} 1 & 1 & 1 \\ 1 & a^2 & a \\ 1 & a & a^2 \end{bmatrix}$$
$$= \begin{bmatrix} Z_S + 3Z_n & 0 & 0 \\ 0 & Z_S & 0 \\ 0 & 0 & Z_S \end{bmatrix}$$

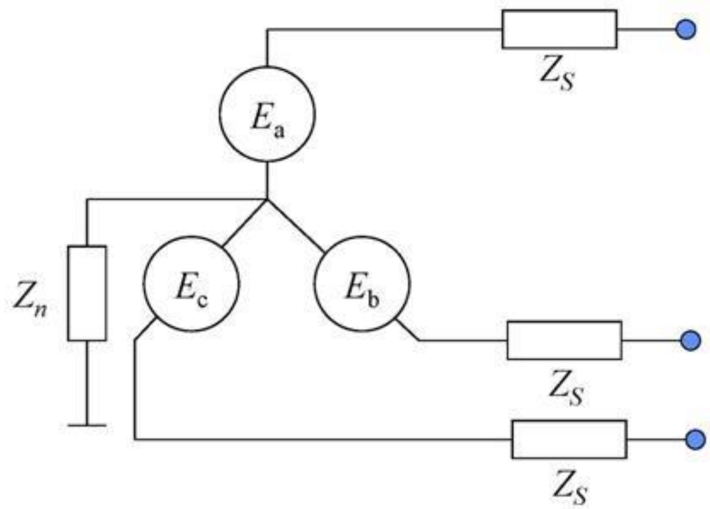
Loaded Generators

Typical values for common generators

- Remember that the transient fault impedance is a function of time.
- Positive sequence values are the same as X_d, X_d', and X_d''.
- Negative sequence values are affected by the rotation of the rotor $X_2 \sim X_d''$
- Zero sequence values are isolated from the airgap of the machine, and the zero sequence reactance is approximated to the leakage reactance

$X_0 \sim X_L$

Figure below represents a three phase synchronous generator with neutral grounded through an impedance Z_n. the generator is supplying a three phase balanced load.



The synchronous machine generates balanced three phase internal voltages and is represented as a positive sequence set of phasors.

$$E_{abc} = \begin{bmatrix} 1 \\ a^2 \\ a \end{bmatrix} E_a$$

The machine is supplying a three phase balanced load. Applying Kirchhoffs voltage law to each phase we obtain

$$V_a = E_a - Z_S I_a - Z_n I_n$$

$$V_b = E_b - Z_S I_b - Z_n I_n$$

$$V_c = E_c - Z_S I_c - Z_n I_n$$

Substituting for $I_n = I_a + I_b + I_c$, we get

$$\begin{bmatrix} V_a \\ V_b \\ V_c \end{bmatrix} = \begin{bmatrix} E_a \\ E_b \\ E_c \end{bmatrix} - \begin{bmatrix} Z_S + Z_n & Z_n & Z_n \\ Z_n & Z_S + Z_n & Z_n \\ Z_n & Z_n & Z_S + Z_n \end{bmatrix} \begin{bmatrix} I_a \\ I_b \\ I_c \end{bmatrix}$$

Or

$$V_{abc} = E_{abc} - Z_{abc} I_{abc}$$

$$AV_{012} = AE_{012} - Z_{abc}AI_{012}$$

Multiplying by A^{-1}, we get

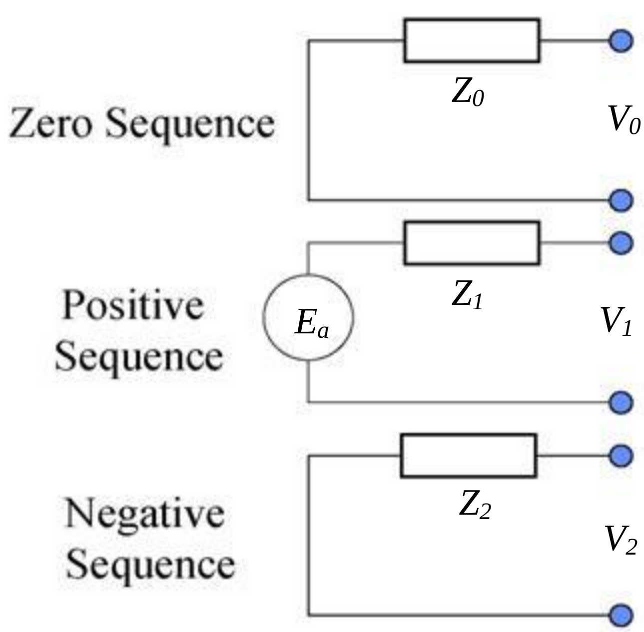
$$V_{012} = E_{012} - A^{-1}Z_{abc}AI_{012} = E_{012} - Z_{012}I_{012}$$

Where

$$Z_{012} = \frac{1}{3}\begin{bmatrix} 1 & 1 & 1 \\ 1 & a & a^2 \\ 1 & a^2 & a \end{bmatrix}\begin{bmatrix} Z_S + Z_n & Z_n & Z_n \\ Z_n & Z_S + Z_n & Z_n \\ Z_n & Z_n & Z_S + Z_n \end{bmatrix}\begin{bmatrix} 1 & 1 & 1 \\ 1 & a^2 & a \\ 1 & a & a^2 \end{bmatrix}$$

$$Z_{012} = \begin{bmatrix} Z_S + 3Z_n & 0 & 0 \\ 0 & Z_S & 0 \\ 0 & 0 & Z_S \end{bmatrix} = \begin{bmatrix} Z_0 & 0 & 0 \\ 0 & Z_1 & 0 \\ 0 & 0 & Z_2 \end{bmatrix}$$

The three equivalent sequence networks are shown below



Transformers

- **Series Leakage Impedance**

  The magnetization current and core losses represented by the shunt branch are neglected (they represent only 1% of the total load current). The transformer is modeled with the equivalent series leakage impedance.

- **Three single-phase units & five-legged core three-phase units**

  The series leakage impedance is the same for all the sequences.

  $Z_0=Z_1=Z_2=Z_L$

- **Three-legged core three-phase units**

  The series leakage impedance is the same for the positive and negative sequence only.

  $Z_1=Z_2=Z_L$

- **Wye-delta transformers create a phase shifting pattern for the various sequences**

  The positive sequence quantities rotate by +30 degrees

  The negative sequence quantities rotate by -30 degrees

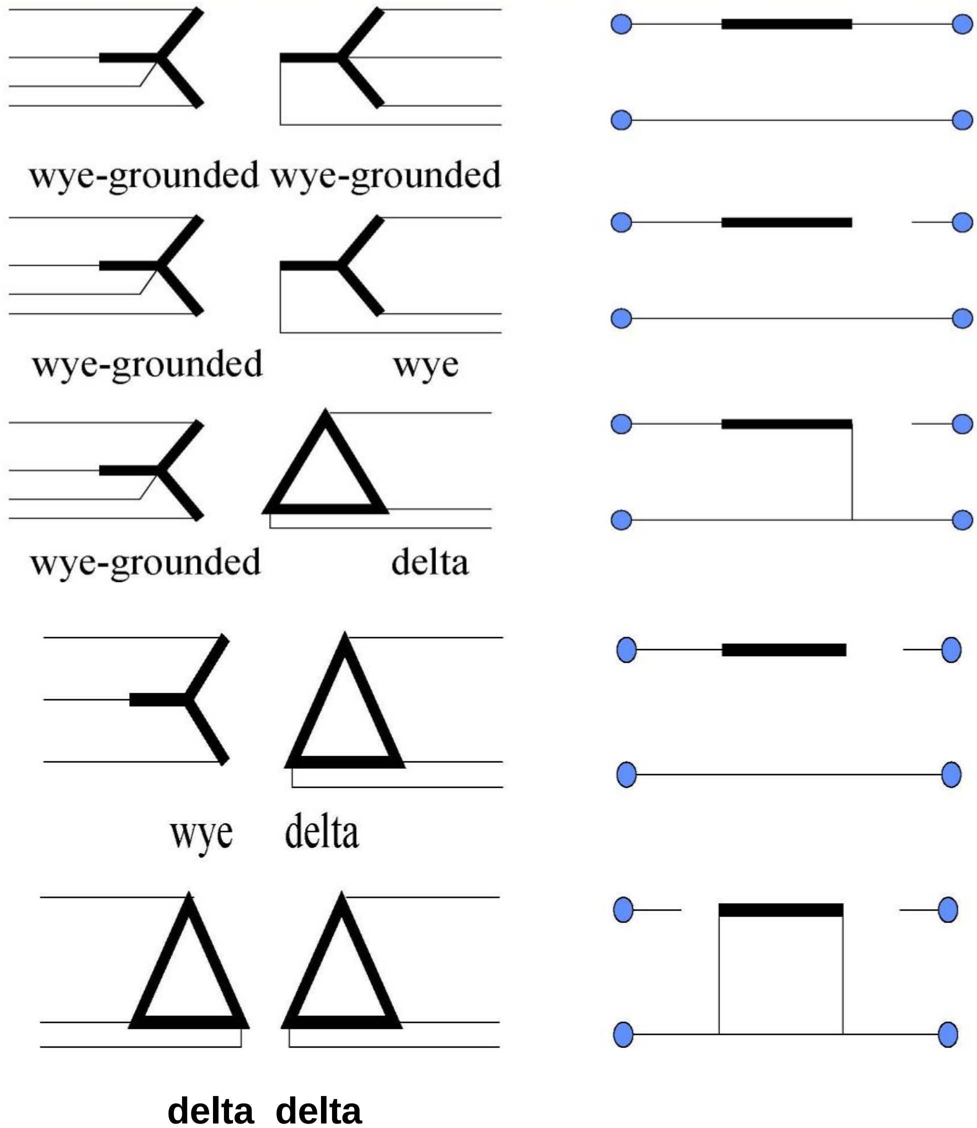
  The zero sequence quantities can not pass through the transformer

- **Zero-sequence network connections of the transformer depends on the winding connection**

  Primary winding - wye / wye-grounded / delta

  Secondary winding - wye / wye-grounded / delta

Zero-sequence network for various connections are shown below:



Reactors

A series reactor is an inductive coil connected in a circuit or system to limit a short circuit current.

Whenever a fault occurs in a system, the circuit breaker interrupts the fault current and isolate the faulty circuit. The circuit breakers in the system are selected so that they can interrupt the maximum short circuit current in the system. However for some systems the short circuit may be so high that it may be worthwhile to limit the current by the use of reactors connected suitably in the circuit. In that case circuit breakers of lesser breaking capacity would be needed. Moreover whenever a system extension takes place, the short circuit current increases. In case it is desired not to place the circuit breakers, the current can be limited by the use of reactors.

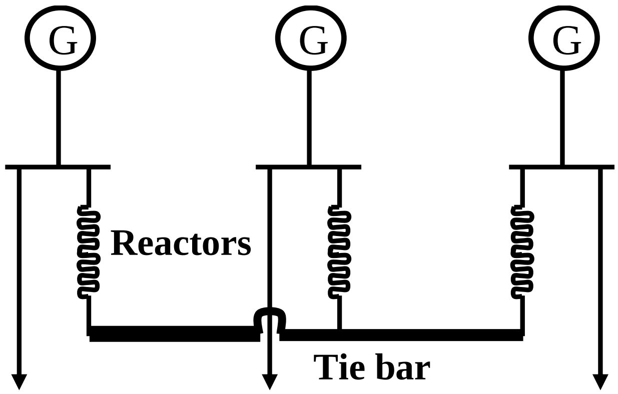
A series reactor may be of fixed or switched type. A fixed reactor is permanently connected in the circuit. A switched reactor is one whose reactance can be varied so that at any time a desired amount of reactance can be connected in the system.

The specifications for a reactor include the type, location (indoor or outdoor), reactance in ohms, kVA rating, rated voltage, rated current and frequency.

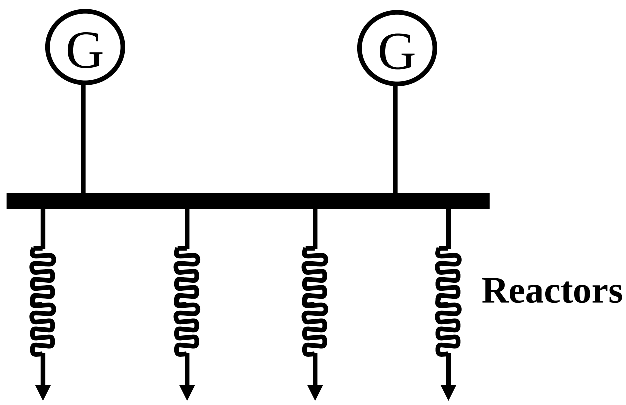
Reactors use now-a-days are mostly air cored coils. They are built in two types, the dry type reactor and oil immersed reactor.

Reactor locations:

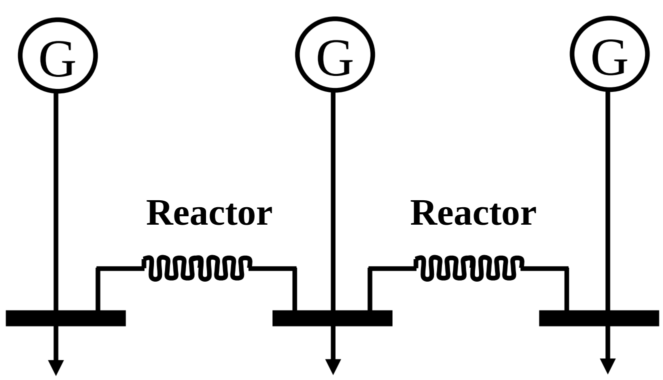
a) Generator Reactor: it is connected in series with the alternator as shown in Figure below. Modern alternators have high inherent reactance and therefore do not need any external reactor for limiting the current. However they are still use for some old machines.



b) Feeder Reactor: A reactor is connected in series with each feeder as shown in Figure below. Since the number of feeders is generally large, this connection entails the use of a large number of reactors. Moreover since the reactors carry full load current, a constant power loss occurs.



c) Bus Bar Reactor: The reactors are inserted in between bus sections. As shown in Figure below. During the normal operations only a small power flows through the reactor and hence no power loss takes place. During short circuit only the generator feeds the fault directly.



Example 6

A three-phase unbalanced source with the following phase-to-neutral voltages

$$V_{abc} = \begin{bmatrix} 300\angle -120^\circ \\ 200\angle 90^\circ \\ 100\angle -30^\circ \end{bmatrix}$$

is applied to the circuit in Figure below. The load series impedance per phase is Z_S = 10 + j40 and the mutual impedance between phases is Z_m = j5. The load and source neutrals are solidly grounded. Determine

(a) The load sequence impedance matrix, $Z_{012} = A^{-1} Z_{abc} A$

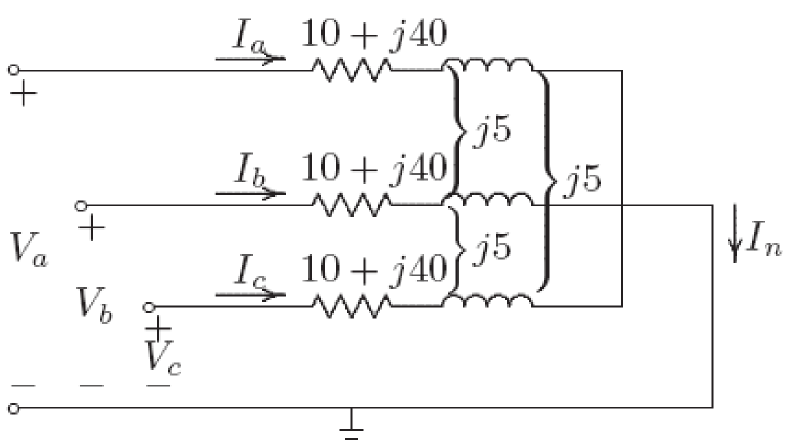
(b) The symmetrical components of voltage.

(c) The symmetrical components of current.

(d) The load phase currents.

(e) The complex power delivered to the load in terms of symmetrical components, $S_{3\phi} = 3(V_0 I_0^* + V_1 I_1^* + V_2 I_2^*)$

(f) The complex power delivered to the load by summing up the power in each phase, $S_{3\phi} = 3(V_a I_a^* + V_b I_b^* + V_c I_c^*)$



Solution:

The results are

$$Z_{012} == \begin{bmatrix} 10 + j50 & 0 & 0 \\ 0 & 10 + j35 & 0 \\ 0 & 0 & 10 + j35 \end{bmatrix}$$

$$V_{012}=\begin{bmatrix} 193.1852\angle -135^\circ \\ 86.9473\angle -84.8961^\circ \end{bmatrix}$$

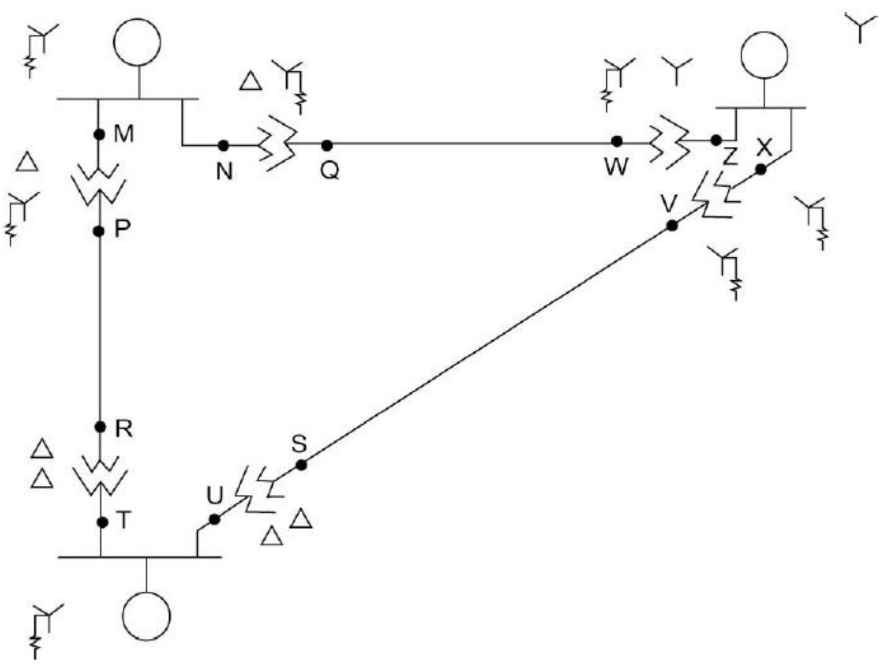
$$I_{012}=\begin{bmatrix} 0.8289\angle 161.3099^\circ \\ 5.3072\angle 150.9454^\circ \\ 2.3886\angle -158.9507^\circ \end{bmatrix}$$

$$I_{abc}=\begin{bmatrix} 7.9070\angle 165.4600^\circ \\ 5.8190\angle 14.8676^\circ \\ 2.7011\angle -96.9315^\circ \end{bmatrix}$$

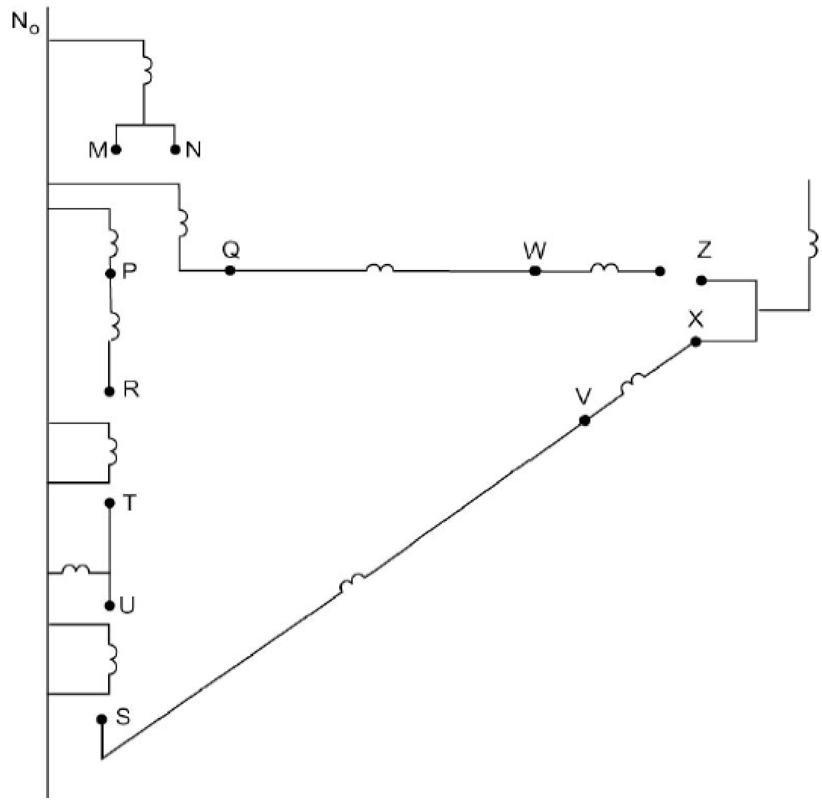
$$S_{3\phi}=1036.8+j3659.6$$

Example 7

Draw the zero sequence network for the system shown in Figure below:



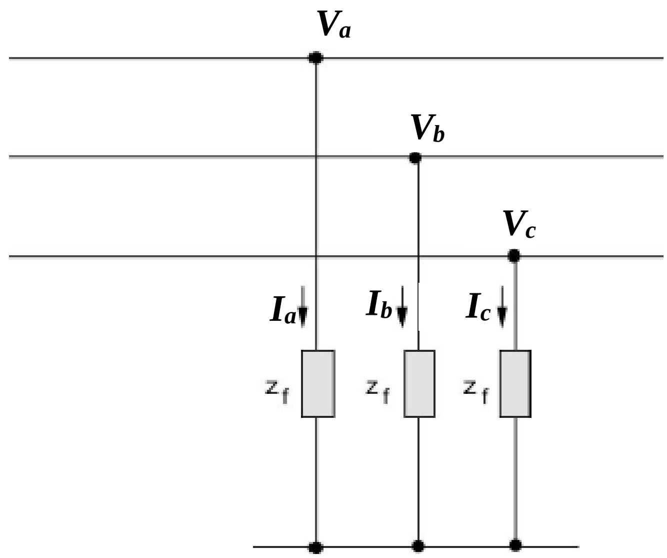
Solution



The Balanced Three-Phase Fault

This type of fault is defined as a simultaneous short circuit across all three phases. It occurs infrequently, but it is the most severe type of fault encountered. Because the network is balanced, it is solved on a per phase basis. The other two phases carry identical currents except for the phase shift.

Let us now consider the situation with a balanced three-phase fault on phases *A, B,* and *C,* all through the same fault impedance Z_f. This fault condition is shown in Figure below.



If the fault impedance is zero, the fault is referred to as the bolted fault or the solid fault. It is clear from inspection in Figure above that the phase voltage at the faults are given by

$$V_a = I_a Z_f$$

$$V_b = I_b Z_f$$

$$V_c = I_c Z_f$$

It can be shown that

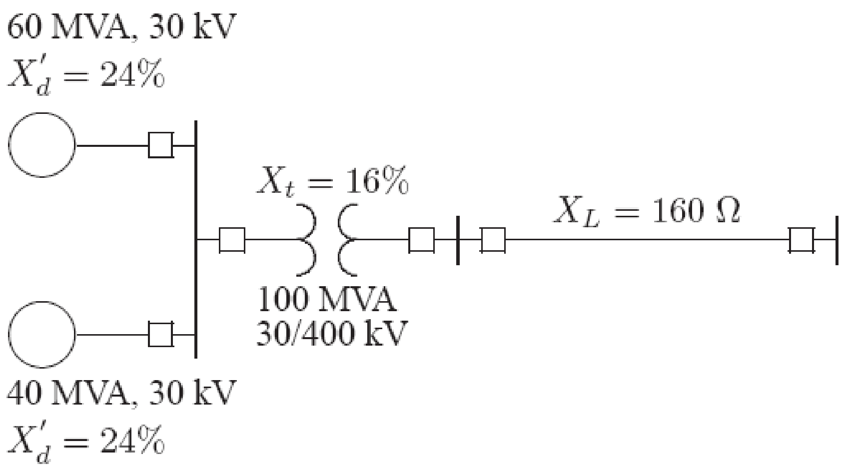
$$I_{a1} = \frac{V_{a1}}{Z_1 + Z_f}$$

$$I_{a2} = I_{a0} = 0$$

The implications of last equation is obvious. No zero sequence nor negative sequence components of the current exist. Instead, only positive sequence quantities are obtained in the case of a balanced three-phase fault.

Example 8

The system shown in Figure below is initially on no load with generators operating at their rated voltage with their emfs in phase. The rating of the generators and the transformers and their respective percent reactances are marked on the diagram. All resistances are neglected. The line impedance is j160Ω. A three-phase balanced fault occurs at the receiving end of the transmission line. Determine the short circuit current and the short-circuit MVA.



Solution:

The base impedance for line is

$$Z_B = \frac{(400)^2}{100} = 1600\Omega$$

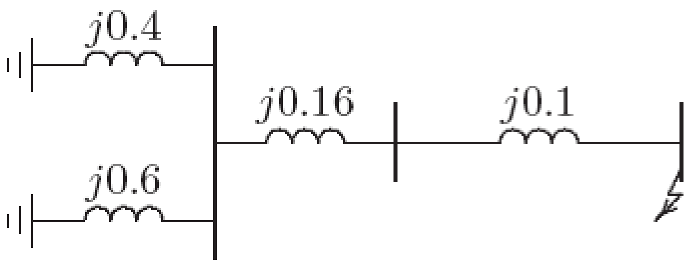
and the base current is

$$I_B = \frac{100,000}{\sqrt{3}(400)} = 144.3375 \text{ A}$$

The reactances on a common 100 MVA base are

$$X'_{dg1} = \frac{100}{60}(0.24) = 0.4 \text{ pu}$$
$$X'_{dg2} = \frac{100}{40}(0.24) = 0.6 \text{ pu}$$
$$X_t = \frac{100}{100}(0.16) = 0.16 \text{ pu}$$
$$X_{line} = \frac{160}{1600} = 0.1 \text{ pu}$$

The impedance diagram is as shown in Figure below



Impedance to the point of fault is

$$X = j\frac{(0.4)(0.6)}{0.4+0.6} + j0.16 + j0.1 = j0.5 \text{ pu}$$

The fault current is

$$\begin{aligned} I_f &= \frac{1}{j0.5} = 2\angle -90^\circ \text{ pu} \\ &= (144.3375)(2\angle -90^\circ) = 288.675\angle -90^\circ \text{ A} \end{aligned}$$

The Short-circuit MVA is

$$\text{SCMVA} = \sqrt{3}(400)(288.675)(10^{-3}) = 200 \text{ MVA}$$

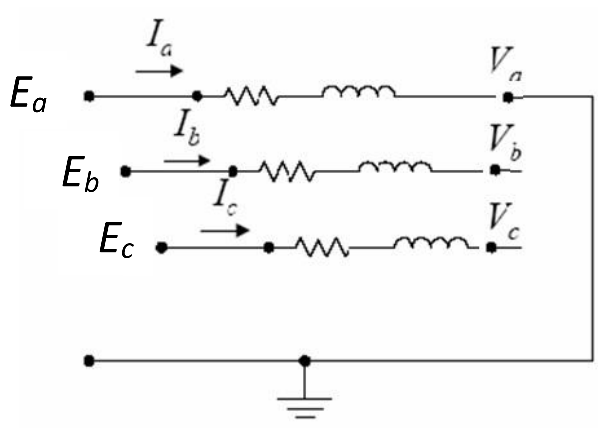
‘

Common Unbalanced Network Faults

- Single-line-to-ground faults (S-L-G)
- Double-line-to-ground faults (L-L-G)
- Line-to-line faults (L-L)

Single Line to Ground Fault (S-L-G)

Assume that phase *A* is shorted to ground at the fault point *F* as shown in Figure below. The phase *B* and *C* currents are assumed negligible, accordingly.



$$I_b = I_c = 0$$

$$\begin{bmatrix} I_{a0} \\ I_{a1} \\ I_{a2} \end{bmatrix} = \frac{1}{3} \begin{bmatrix} 1 & 1 & 1 \\ 1 & a & a^2 \\ 1 & a^2 & a \end{bmatrix} \begin{bmatrix} I_a \\ 0 \\ 0 \end{bmatrix}$$

$$I_{ao} = I_{a1} = I_{a2} = 1/3 I_a$$

$$V_a = V_{a0} + V_{a1} + V_{a2} = 0$$

$$V_a = E_a - Z_a I_a = 0$$

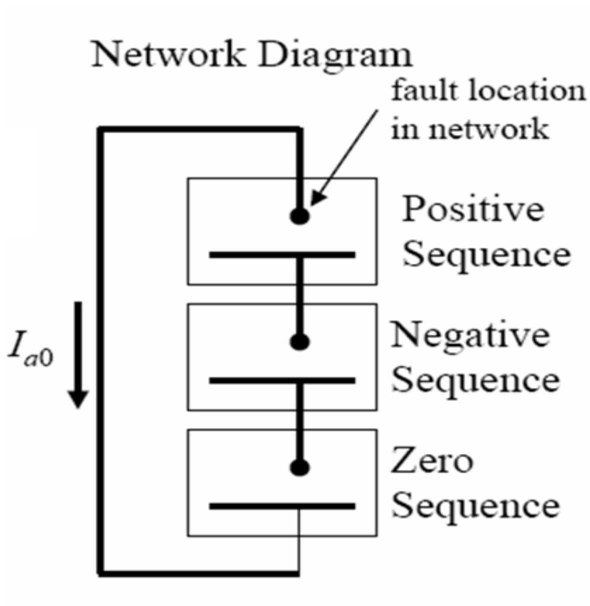
$$V_a = E_a - (Z_{012} I_{012})$$

$$V_a = E_a - (Z_{ao} I_{ao} + Z_{a1} I_{a1} + Z_{a2} I_{a2})$$

$$V_a = E_a - (Z_{ao} + Z_{a1} + Z_{a2}) I_{ao}$$

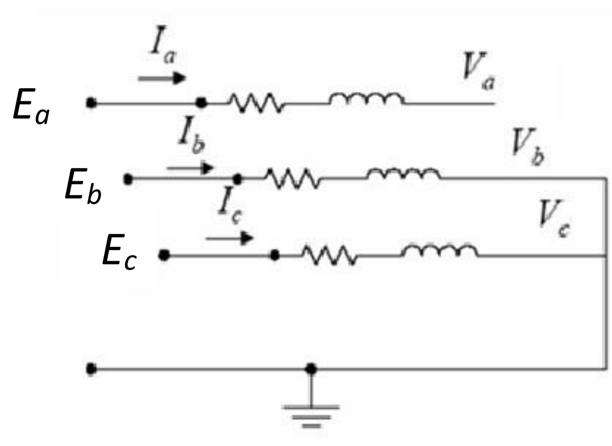
$$I_{ao} = \frac{E_a}{(Z_{ao} + Z_{a1} + Z_{a2})} \quad \Longrightarrow \quad \boxed{I_f = 3 I_{ao}}$$

The resulting equivalent circuit is shown in Figure below.



Double-line-to-ground faults (L-L-G)

Assume that phase B and phase C is shorted to ground as shown in Figure below.



$$V_b = 0$$

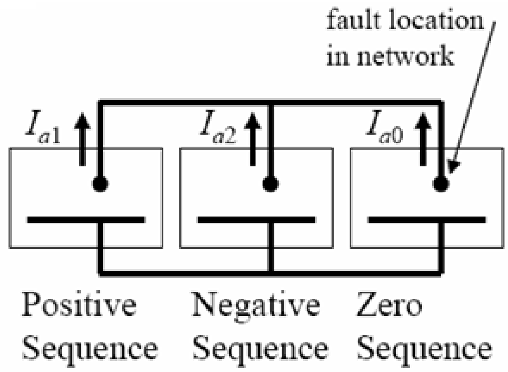
$$V_c = 0$$

$$I_a = I_{ao} + I_{a1} + I_{a2} = 0$$

$$I_f = I_b + I_c$$

$$\begin{bmatrix} V_{a0} \\ V_{a1} \\ V_{a2} \end{bmatrix} = \frac{1}{3} \begin{bmatrix} 1 & 1 & 1 \\ 1 & a & a^2 \\ 1 & a^2 & a \end{bmatrix} \begin{bmatrix} V_a \\ 0 \\ 0 \end{bmatrix}$$

$$V_{a0} = V_{a1} = V_{a2}$$



$$I_{a1} = V_{a1}\frac{Z_2 + Z_0}{Z_2 Z_0}$$

$$I_{a1} = \frac{E_a(Z_2 + Z_0)}{(Z_1 Z_0 + Z_2 Z_0 + Z_1 Z_1)}$$

$$I_{a1} = \frac{E_a}{(Z_1 + \frac{Z_2 Z_0}{Z_2 + Z_0})}$$

$$I_{a0} = -\frac{V_{a0}}{Z_0} \quad \& \quad I_{a2} = -\frac{V_{a2}}{Z_2}$$

$$I_{abc} = AI_{012}$$

Since $I_{ao} + I_{a1} + I_{a2} = 0$ & $I_f = I_b + I_c$

$$\therefore I_f = I_{ao} + a^2 I_{a1} + aI_{a2} + I_{ao} + aI_{a1} + a^2 I_{a2}$$

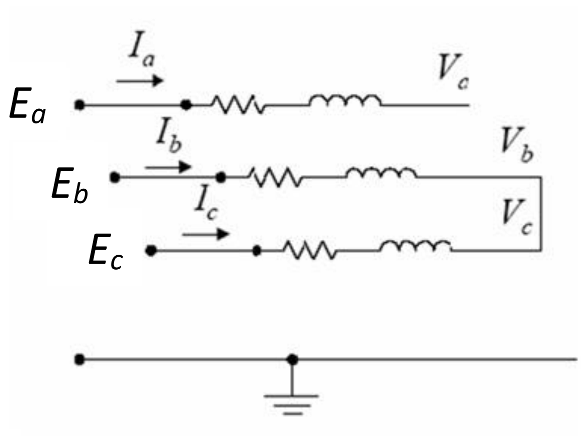
$$= 2I_{ao} + a^2(I_{a1} + I_{a2}) + a(I_{a1} + I_{a2}) = 2I_{ao} + (a^2 + a)(I_{a1} + I_{a2})$$

$$(a^2 + a) = -1$$

$$\Longrightarrow \quad \boxed{I_f = 2I_{ao} - (I_{a1} + I_{a2})}$$

Line-to-line faults (L-L)

Let phase ***A*** be the unfaulted phase. Figure below shows a three-phase system with a line-to-line short circuit between phases *B* and *C*.



$$V_b - V_c = 0$$

$$I_b + I_c = 0$$

$$I_a = I_{a0} + I_{a1} + I_{a2} = 0$$

$$I_f = I_b - I_c$$

$$\begin{bmatrix} I_{a0} \\ I_{a1} \\ I_{a2} \end{bmatrix} = \frac{1}{3}\begin{bmatrix} 1 & 1 & 1 \\ 1 & a & a^2 \\ 1 & a^2 & a \end{bmatrix}\begin{bmatrix} 0 \\ I_b \\ -I_b \end{bmatrix}$$

$$I_{a0} = 0$$

$$I_{a1} = \frac{1}{3}(a - a^2)I_b$$

$$I_{a2} = \frac{1}{3}(a^2 - a)I_b$$

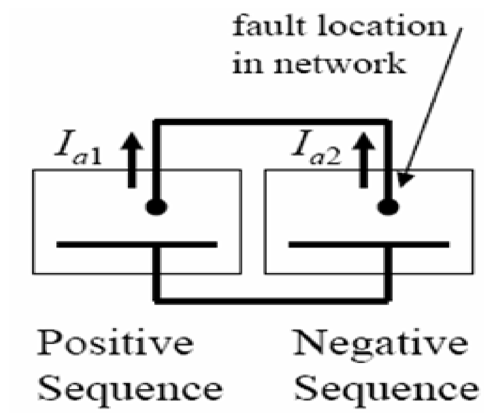
$$I_{a1} = -I_{a2}$$

$$V_b - V_c = (a^2 - a)(V_{a1} - V_{a2}) = 0$$

$$V_{a1} = V_a - Z_1 I_{a1}$$

$$V_{a2} = -Z_2 I_{a2} = Z_2 I_{a1}$$

$$(a^2 - a)\left[V_a - (Z_1 + Z_2)I_{a1}\right] = 0$$



$$V_a - (Z_1 + Z_2) I_{a1} = 0$$

$$I_{a1} = \frac{V_{a(pre-f)}}{Z_1 + Z_2}$$

$$I_{a1} = -I_{a2}$$

$$\begin{bmatrix} I_a \\ I_b \\ I_c \end{bmatrix} = \begin{bmatrix} 1 & 1 & 1 \\ 1 & a^2 & a \\ 1 & a & a^2 \end{bmatrix} \begin{bmatrix} 0 \\ I_{a1} \\ -I_{a1} \end{bmatrix}$$

$$I_b = (a^2 - a) I_{a1} = -j\sqrt{3} I_{a1}$$

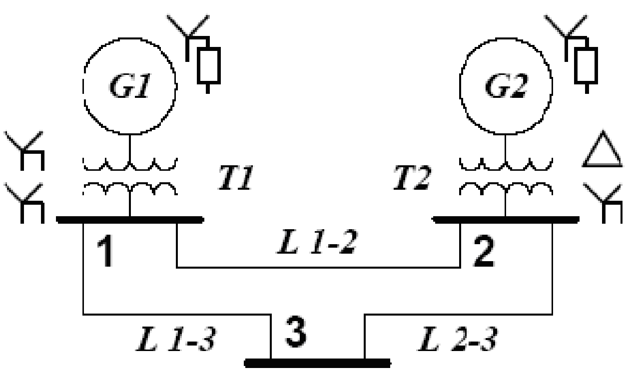
$$I_b = \frac{-j\sqrt{3} V_a}{Z_1 + Z_2}$$

$$I_c = -I_b$$

Example 9

The neutral of each generator is grounded through a current limiting resistor of 8.333% on a 100MVA base. The generators are running at no-load at rated voltage and in phase, all network data is expressed on a 100MVA base.

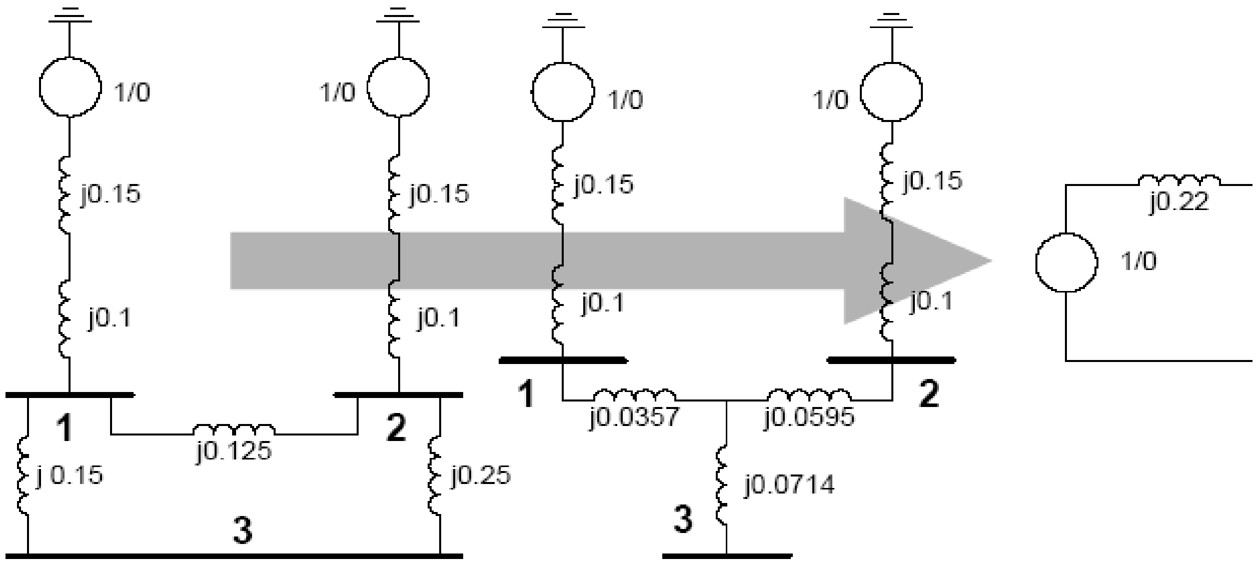
Find : fault current for 3-phase, 1-phase. L-L, L-L-G.



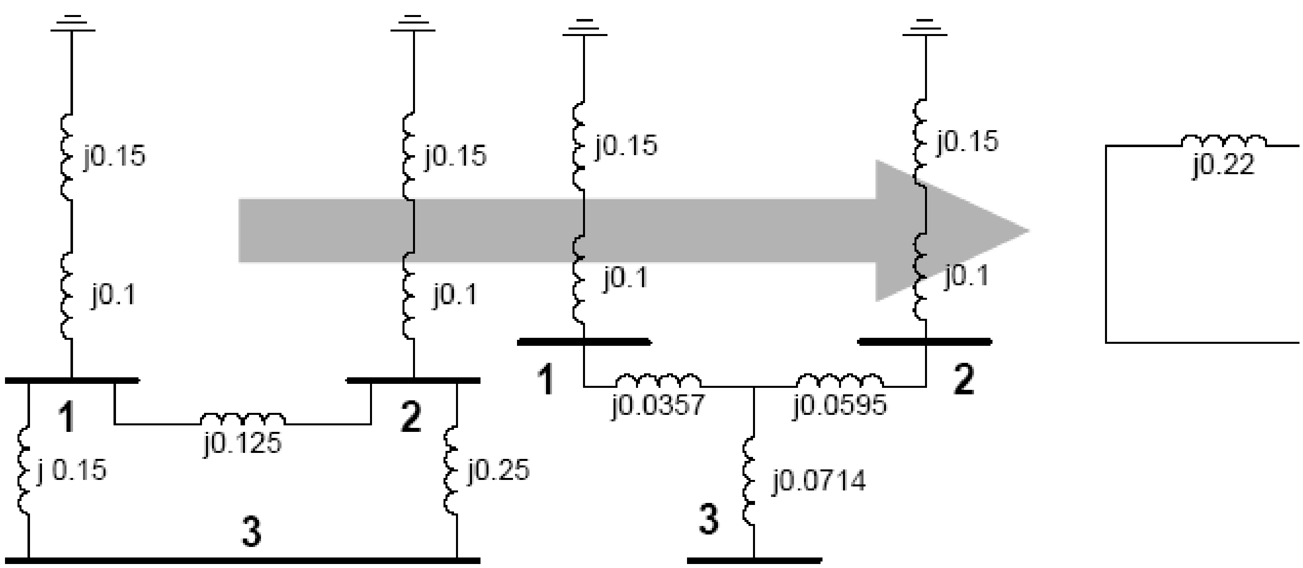
Item	kV Rating	% X_1	% X_2	% X_0
G1	20	15	15	5
G2	20	15	15	5
T1	20/200	10	10	10
T2	20/200	10	10	10
L1-2	200	12.5	12.5	30
L1-3	200	15	15	35
L2-3	200	25	25	71.25

Solution:

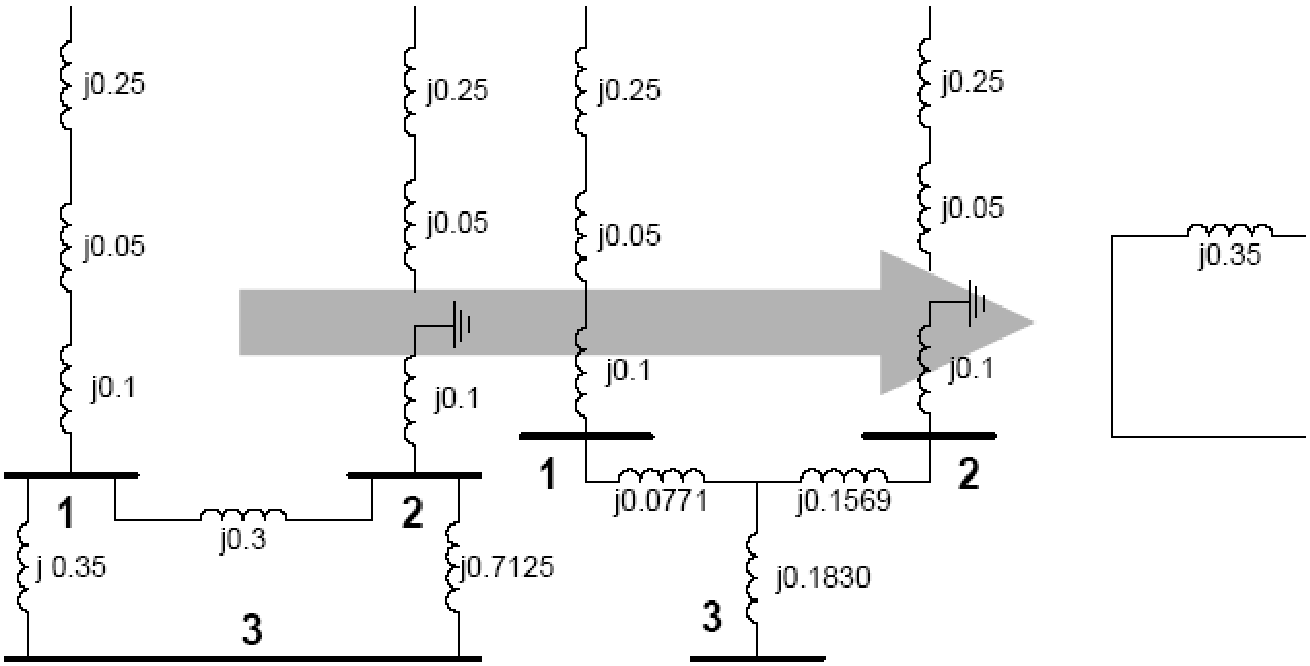
- **Positive Sequence Network**



- **Negative Sequence Network**



- **Zero Sequence Network**



- **3-phase fault**

$$I_{a1} = \frac{V_{a1}}{Z_1} = \frac{1\angle 0^\circ}{j0.22} = -j4.54 p.u.$$

$$I_{af} = I_{a1} = -j4.54 p.u.$$

- **Single line - to- ground (S-L-G)**

$$I_{ao} = I_{a1} = I_{a2} = \frac{E_a}{(Z_{ao} + Z_{a1} + Z_{a2})}$$

$$= \frac{1.0\angle 0^\circ}{j(0.35 + 0.22 + 0.22)} = -j1.266 p.u.$$

$$I_{af} = 3I_{a0} = -j3.80 p.u.$$

• Line-Line fault (L-L)

$$I_{a0} = 0$$

$$I_{a1} = -I_{a2} = \frac{V_{a1}}{Z_1 + Z_2} = \frac{1\angle 0^\circ}{j(0.22 + 0.22)} = -j2.27\,p.u.$$

$$I_{bf} = -I_{cf} = -j\sqrt{3}(-j2.27) = -3.936\,p.u.$$

• Double line - to- ground (D-L-G)

$$I_{a1} = \frac{V_{a1}}{(Z_1 + \frac{Z_2 Z_0}{Z_2 + Z_0})} = \frac{1.0\angle 0^\circ}{j0.22 + \frac{j0.35 * j0.22}{j0.35 + j0.22}} = -j2.816\,p.u.$$

$$I_{a2} = -\frac{V_{a1} - Z_1 I_{a1}}{Z_2} = -\frac{1.0\angle 0^\circ - (j0.22) * (-j2.816)}{j0.22} = j1.729\,p.u.$$

$$I_{a0} = -\frac{V_{a1} - Z_1 I_{a1}}{Z_0} = -\frac{1.0\angle 0^\circ - (j0.22) * (-j2.816)}{j0.35} = j1.087\,p.u.$$

$$I_f = I_b + I_c = 2I_{ao} + (a^2 + a)(I_{a1} + I_{a2})$$

$$= 2(j1.087) - (-j2.816 + j1.729) = j3.261\,p.u.$$

Example 10

Draw the positive, negative and zero sequence networks for the following at bus 2.

(a) Balanced three-phase

(b) Single phase-to-ground

(c) Phase–to-phase

(d) Double phase-to-ground faults for the following system.

Transmission Line Data

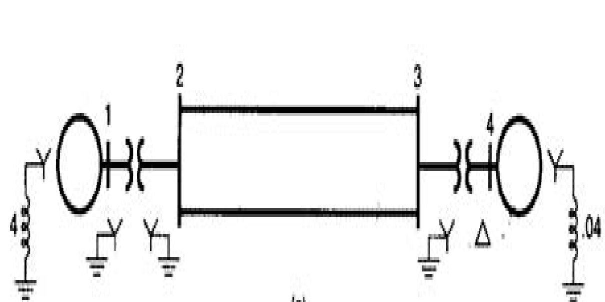
Line	**Bus**	**Bus**	**Seq**	**R**	**X**	**B**	**Srat**
1	2	3	pos	0.00000	0.16000	0.00000	1.0000
			zero	0.00000	0.50000	0.00000	
2	2	3	pos	0.00000	0.16000	0.00000	1.0000
			zero	0.00000	0.50000	0.00000	

Transformer Data

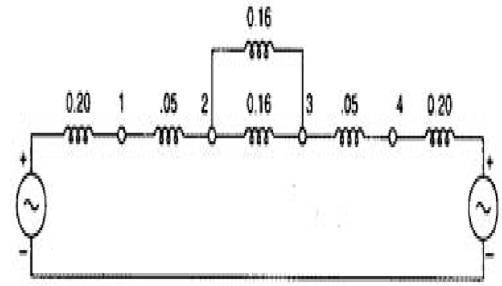
Trans.	**HV Bus**	**LV Bus**	**Seq**	**R**	**X**	**C**	**Srat**
1	2	1	pos	0.00000	0.05000	1.00000	1.0000
	Y	Y	zero	0.00000	0.05000		
2	3	4	pos	0.00000	0.05000	1.00000	1.0000
	Y	D	zero	0.00000	0.05000		

Generator Data

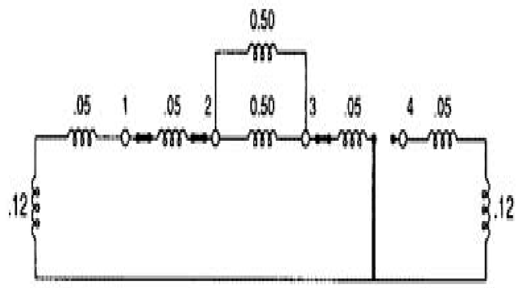
No.	**Bus**	**Srated**	**Ra**	**Xd''**	**Xo**	**Rn**	**Xn**	**Con**
1	1	1.0000	0.0000	0.200	0.0500	0.0000	0.0400	Y
2	4	1.0000	0.0000	0.200	0.0500	0.0000	0.0400	Y



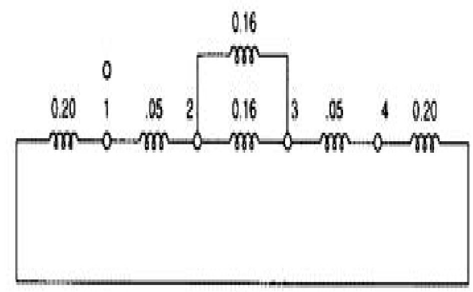
(a)



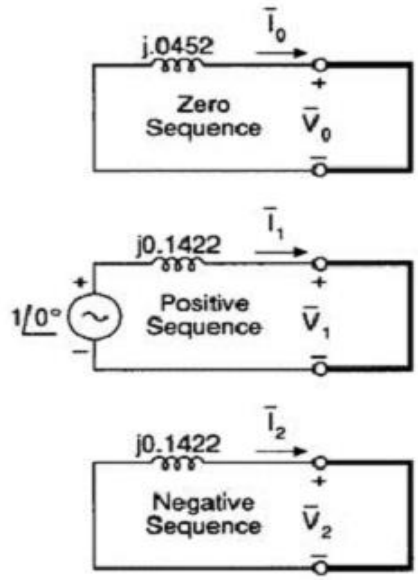
+ ve sequence



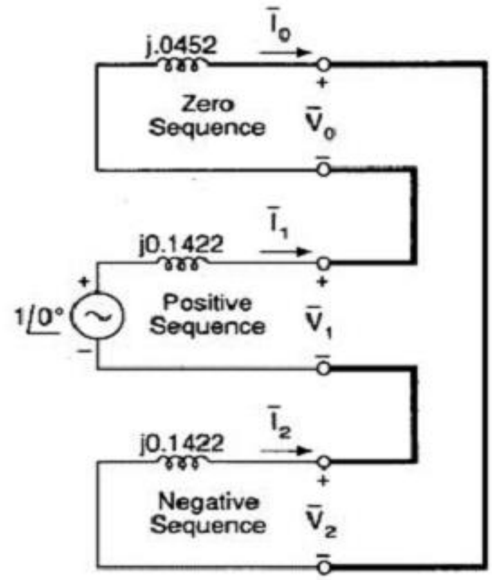
zero sequence



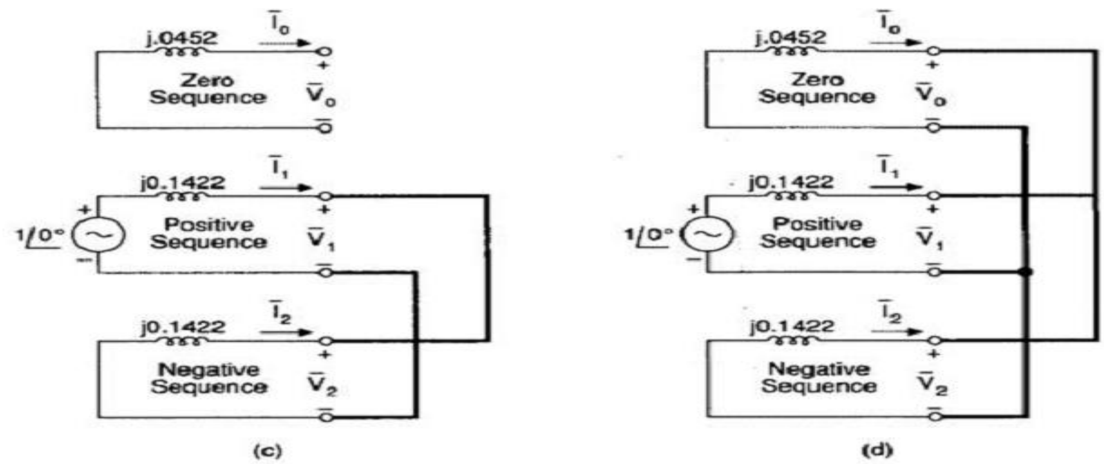
- ve sequence



(a)

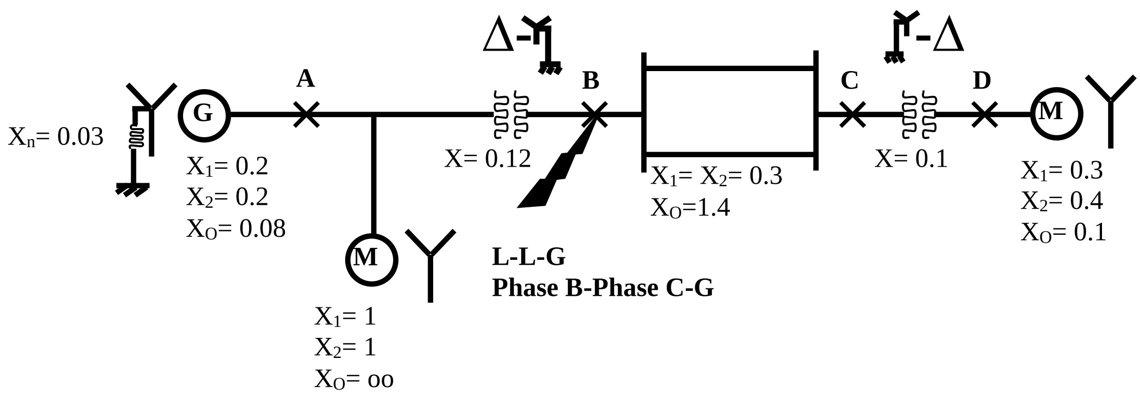


(b)



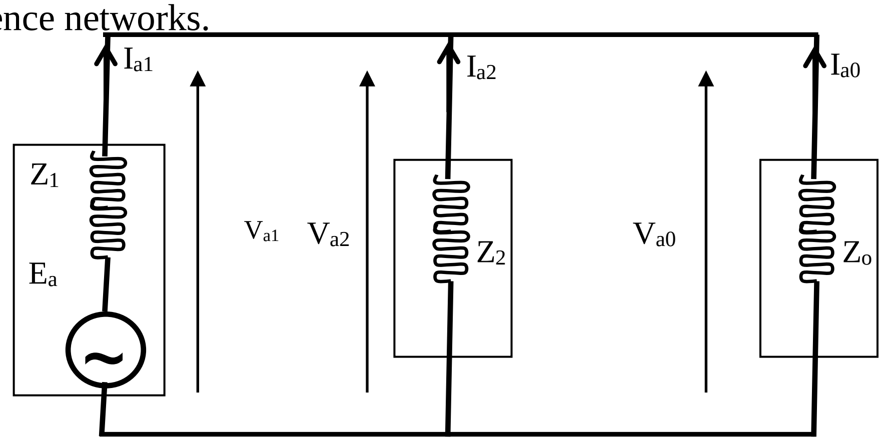
Example 11

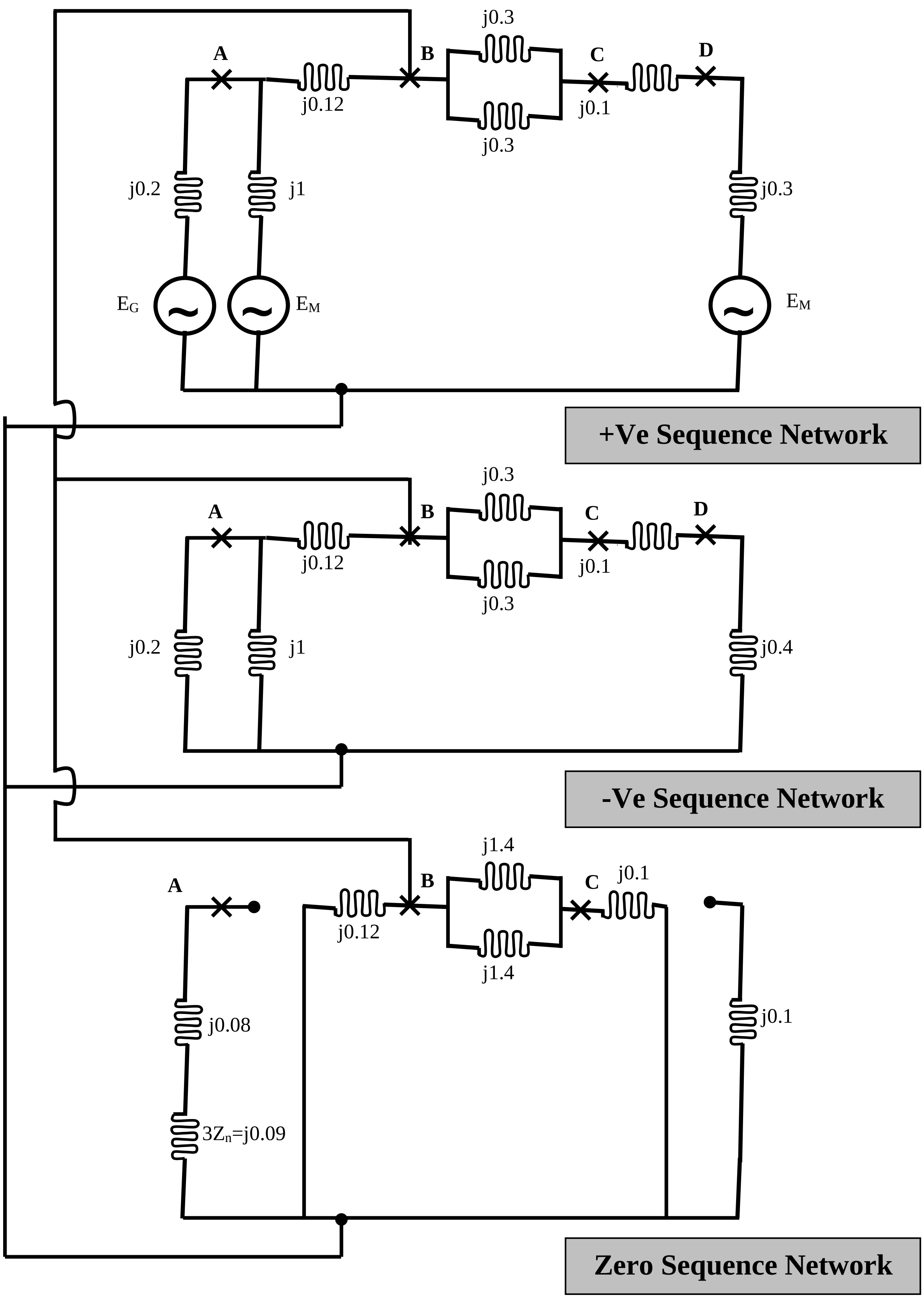
A L-L-G fault occurs at point B in the system shown below. Determine the voltages and currents at the fault



Solution:

A L-L-G is represented by the parallel connection of the +Ve, -Ve and Zero sequence networks.




j0.3
A
B
C
D
j0.12
j0.1
j0.3
j0.2
j1
j0.3
EG
EM
EM
+Ve Sequence Network
j0.3
A
B
C
D
j0.12
j0.1
j0.3
j0.2
j1
j0.4
-Ve Sequence Network
j1.4
A
B
C
j0.1
j0.12
j1.4
j0.08
j0.1
3Zn=j0.09
Zero Sequence Network

$$Z_1 = \frac{j((\frac{0.3}{2}) + 0.1 + 0.3)(\frac{1*0.2}{1+0.2} + 0.12)}{(\frac{0.3}{2} + 0.1 + 0.3) + (\frac{1*0.2}{1+0.2} + 0.12)} =$$

$$Z_2 = \frac{j((\frac{0.3}{2}) + 0.1 + 0.4)(\frac{1*0.2}{1+0.2} + 0.12)}{(\frac{0.3}{2} + 0.1 + 0.4) + (\frac{1*0.2}{1+0.2} + 0.12)} =$$

$$Z_o = \frac{j(0.12)(\frac{0.14}{2} + 0.1)}{(0.12 + \frac{0.14}{2} + 0.1)} =$$

$$I_{a1} = \frac{E_a}{Z_1 + \frac{Z_2 Z_O}{Z_2 + Z_O}} = 3.9\,p.u.$$

$$I_{a2} = -I_{a1}\frac{Z_O}{Z_2 + Z_O} = -1.34\,p.u.$$

$$I_{ao} = -I_{a1} - I_{a2} = -2.56\,p.u.$$

$$V_{ao} = V_{a1} = V_{a2} = -I_{a2}Z_2 = j0.267\,p.u.$$

$$I_a = I_{ao} + I_{a1} + I_{a2} = 0$$

$$I_b = I_{ao} + a^2 I_{a1} + aI_{a2} = (-3.84 - j4.54)\,p.u.$$

$$I_c = I_{ao} + aI_{a1} + a^2 I_{a2} = (-3.84 + j4.54)\,p.u.$$

Ground current $I_g = I_a + I_b + I_c = 3I_{ao} = -7.68$ p.u.

$V_a = V_{ao} + V_{a1} + V_{a2} = 3 * j0.267 = j0.801$ p.u.

$V_b = V_c = 0$

Example 12

Consider a system with sequence impedances given by $Z_1 = j0.2577$, $Z_2 = j0.2085$, and $Z_0 = j0.14$; find the voltages and currents at the fault point for a single line-to-ground fault.

Solution:

The sequence currents are given by

$$I_0 = I_1 = I_2 = \frac{1}{j(0.2577 + 0.2085 + 0.14)}$$

$$= 1.65\angle -90^\circ \; p.u.$$

Therefore

$$I_a = 3I_0 = 4.95\angle -90^\circ \; p.u.$$

$$I_b = I_c = 0$$

The sequence voltages are as follows:

$$V_1 = E_{a1} - I_1 Z_1$$

$$= 1\angle 0^\circ - (1.65\angle -90^\circ)(0.2577\angle 90^\circ)$$

$$= 0.57 \, p.u.$$

$$V_2 = -I_2 Z_2$$

$$= -(1.65\angle -90^\circ)(0.2085\angle 90^\circ)$$

$$= -0.34 \, p.u.$$

$$V_0 = -I_0 Z_0$$

$$= -(1.65\angle -90^\circ)(0.14\angle 90^\circ)$$

$$= -0.23 p.u.$$

The phase voltages are thus

$$V_a = V_{a0} + V_{a1} + V_{a2} = 0$$

$$V_b = a^2 V_{a0} + aV_{a1} + V_{a2}$$

$$= (1\angle 240^\circ)(0.57) + (1\angle 120^\circ)(-0.34) + (-0.23)$$

$$= 0.86\angle -113.64^\circ \; p.u.$$

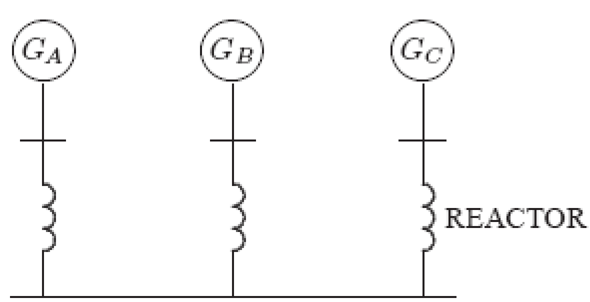
$$V_c = aV_{a0} + a^2 V_{a1} + V_{a2}$$

$$= (1\angle 120^\circ)(0.57) + (1\angle 240^\circ)(-0.34) + (-0.23)$$

$$= 0.86\angle 113.64^\circ \; p.u.$$

<u>Example 13</u>

Three 15-MVA, 30-kV synchronous generators A, B, and C are connected via three reactors to a common bus bar, as shown in Figure below. The neutrals of generators A and B are solidly grounded, and the neutral of generator C is grounded through a reactor of 2.0 Ω. The generator data and the reactance of the reactors are tabulated below. A line-to-ground fault occurs on phase *a* of the common bus bar. Neglect prefault currents and assume generators are operating at their rated voltage. Determine the fault current in phase *a*.



Item	X^1	X^2	X^0
G_A	0.25 pu	0.155 pu	0.056 pu
G_B	0.20 pu	0.155 pu	0.056 pu
G_C	0.20 pu	0.155 pu	0.060 pu
Reactor	6.0 Ω	6.0 Ω	6.0 Ω

Solution:

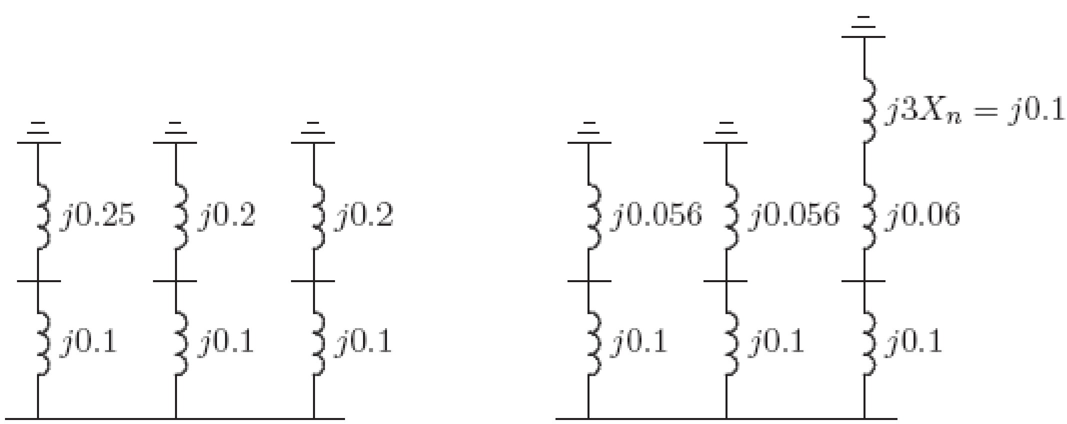
The generator base impedance is

$$Z_B = \frac{(30)^2}{15} = 60 \;\; \Omega$$

The reactor per-unit reactance, and the per-unit generator C neutral reactor are

$$\begin{aligned} X_R &= \frac{6}{60} = 0.1 \;\; \text{pu} \\ X_n &= \frac{2}{60} = 0.3333 \;\; \text{pu} \end{aligned}$$

The positive-sequence impedance network, and the zero sequence impedance network is shown in Figure below.



(a) Positive sequence network (b) Zero sequence network

The positive-sequence impedance is

$$\frac{1}{X^1} = \frac{1}{0.35} + \frac{1}{0.3} + \frac{1}{0.3} \quad \text{or} \quad X^1 = 0.105$$

The negative-sequence impedance network is the same as the positive-sequence impedance network, except for the value of the generator negative-sequence reactance. Therefore, the negative-sequence impedance is

$$\frac{1}{X^2} = \frac{1}{0.255} + \frac{1}{0.255} + \frac{1}{0.255} \quad \text{or} \quad X^2 = 0.085$$

The zero-sequence impedance is

$$\frac{1}{X^0} = \frac{1}{0.156} + \frac{1}{0.156} + \frac{1}{0.26} \quad \text{or} \quad X^0 = 0.06$$

The line-to-ground fault current in phase *a* is

$$I_a = 3I_a^0 = \frac{3(1)}{j(0.105 + 0.085 + 0.06)} = 12\angle{-90^\circ} \text{ pu}$$

<u>Example 14</u>

Repeat example 15 for a bolted line-to-line fault between phases *b* and *c*.

Solution:

The positive-sequence fault current in phase *a* is

$$I_a^1 = \frac{1}{Z^1 + Z^2} = \frac{1}{j(0.105 + 0.085)} = -j5.26316 \text{ pu}$$

The fault current is

$$I_b = -j\sqrt{3}I_a^1 = -9.116 \text{ pu}$$

<u>Example 15</u>

Repeat example 15 for a bolted double line-to-ground fault on phases *b* and *c*.

Solution:

The positive- and zero-sequence fault currents in phase *a* are

$$I_a^1 = \frac{1}{j0.105 + j\left(\frac{(0.085)(0.06)}{0.085+0.06}\right)} = -j7.13407 \text{ pu}$$

$$I_a^0 = -\frac{1 - (j0.105)(-j7.13407)}{j0.06} = j4.182 \text{ pu}$$

The fault current is

$$I_f = 3I_a^0 = 12.546\angle 90^\circ$$