Lec. 5.

Per-Unit (*pu*) Representation

4.1 Introduction

The per-unit (*pu*) system is used extensively in power system calculations. The representation simplifies the vast scaling of sizes from super-large generation and transmission networks to the industrial distribution system or a residential load. The definition of the per-unit value of any quantity is given as:

$$pu\ value = \frac{actual\ value}{base\ value\ of\ the\ same\ dimension} \tag{1}$$

In any electrical network, a minimum of four base quantities is required to define completely a per-unit system: voltage, current, power, and impedance (or admittance).

$$per\ unit\ power = \frac{actual\ power}{base\ power} \tag{2}$$

$$per\ unit\ voltage = \frac{actual\ voltage}{base\ voltage} \tag{3}$$

$$per\ unit\ current = \frac{actual\ current}{base\ current} \tag{4}$$

$$per\ unit\ impedance = \frac{actual\ impedance}{base\ impedance} \tag{5}$$

If any two of these quantities are chosen arbitrarily, the other two become fixed. For example, selecting base values for voltage and power fixes the base values for current and impedance. Therefore, on a per phase basis the following relationships hold:

$$base\ current = \frac{base\ power}{base\ voltage} \tag{6}$$

$$base\ impedance = \frac{base\ voltage}{base\ current} \tag{7}$$

<u>EXAMPLE 4.1</u>

Calculate the base impedance and base current for a single-phase system if the base voltage is 7.2 kV and the base apparent power is 10 MVA.

$$Z_{Base} = \frac{(V_{Base})^2}{S_{Base}} = \frac{(7200\text{ V})^2}{10{,}000{,}000\text{ VA}}$$
$$Z_{Base} = 5.184\ \Omega$$
$$I_{Base} = \frac{S_{Base}}{V_{Base}} = \frac{10{,}000{,}000\text{ VA}}{7200\text{ V}}$$
$$I_{Base} = 1388.9\text{ A}$$

For three-phase systems, we use the total or three-phase power and the line-to-line voltage as the base. For currents and impedance values, it is common practice to convert the system to a wye-connected network, and use the phase current and phase impedance as the bases. Hence, the base impedance and the base current can be computed directly from the three-phase values of the base apparent power and the line-to-line base voltage. Delta-connections for the moment are converted to wye-connected equivalents.

Recall for wye-connections, $V_{LL}= \sqrt{3}V_p$, and from Equations 8 and 7 we find:

$$S_{3\phi} = 3 \cdot V_p \cdot I_p^* \tag{8}$$
$$= \sqrt{3} \cdot V_L \cdot I_L^*$$

$$base \quad current, \quad I_{Base} = \frac{S_{1\phi-Base}}{V_{p-Base}} = \frac{S_{3\phi-Base}/3}{V_{LL-Base}/\sqrt{3}} = \frac{S_{3\phi-Base}}{\sqrt{3}\cdot V_{LL-Base}}$$

$$Base \quad impedance, \quad Z_{Base} = \frac{V_{p-Base}}{I_{Base}} = \frac{V_{LL-Base}/\sqrt{3}}{I_{Base}}$$

and

$$Z_{Base} = \frac{\left(V_{p-Base}\right)^2}{S_{1\phi-Base}} = \frac{\left(V_{LL-Base}/\sqrt{3}\right)^2}{S_{3\phi-Base}/3} = \frac{\left(V_{LL-Base}\right)^2}{S_{3\phi-Base}}$$

<u>EXAMPLE 4.2</u>

A three-phase system delivers 18,000 kW to a pure resistive wye-connected load. The line-to-line voltage at the load terminals is 108 kV. Assuming the three-phase power base is 30,000 kVA and the voltage base is 120 kV, find the following per unit quantities for the load:

a) the per unit voltage,

b) the per unit power,

c) the per unit current, and

d) the per unit impedance.

a) The line-to-line base voltage is:

$V_{LL\text{-}Base}$= 120Kv

And the phase base voltage is:

$$V_{p-Base} = \frac{V_{LL-Base}}{\sqrt{3}}$$

$$= \frac{120kV}{\sqrt{3}} = 69.2kV$$

The actual line-to-neutral voltage is:

$$V_p = \frac{V_{LL}}{\sqrt{3}}$$

$$= \frac{108kV}{\sqrt{3}} = 62.4kV$$

The per unit voltage is:

$$V_{pu} = \frac{V_p}{V_{p-Base}} = \frac{V_{LL}}{V_{LL-Base}}$$

$$= \frac{62.4kV}{69.2kV} = \frac{108kV}{120kV} = 0.900\,pu$$

b) The three-phase base power is:

$S_{3\varphi}$= 30,000kVA

and the single-phase base power is:

$$S_{1\phi} = \tfrac{1}{3} S_{3\phi}$$

$$= \frac{30{,}000kVA}{3} = 10{,}000kVA$$

The per unit power is:

$$P_{pu} = \frac{P_{1\phi}}{S_{1\phi-Base}} = \frac{P_{3\phi}}{S_{3\phi-Base}}$$

$$= \frac{6000W}{10{,}000VA} = \frac{18{,}000W}{30{,}000VA} = 0.600\,pu$$

c) The per unit current is:

$$I_{pu} = \left(\frac{S_{pu}}{V_{pu}}\right)^*$$

$$= \left(\frac{0.600 + j0.0\,pu}{0.900\,pu}\right)^* = 0.667\,pu$$

The base current is:

$$I_{Base} = \frac{S_{3\phi-Base}}{\sqrt{3}\cdot V_{LL-Base}}$$
$$= \frac{30{,}000kVA}{\sqrt{3}\cdot(120kV)} = 144.3A$$

To verify the results, first calculate the actual current, which is:

$$I_p = I_{Base}\cdot I_{pu}$$
$$= (144.3A)(0.667pu) = 96.2A$$

Calculate the load current from actual voltage and power:

$$\hat{I}_p = \frac{6{,}000kVA}{62.4kV} = 96.2A$$
$$\hat{I}_p = I_p = 96.2A$$

d) The per unit impedance is:

$$Z_{pu} = \frac{V_{pu}}{I_{pu}}$$
$$= \frac{0.900pu}{0.667pu} = 1.350 + j0.0pu$$

The base impedance is:

$$Z_{Base} = \frac{V_{1\phi-Base}}{I_{Base}} = \frac{(V_{3\phi-Base})^2}{S_{3\phi-Base}}$$
$$= \frac{(120kV)^2}{30MVA} = 480\Omega$$

To verify the results, first calculate the actual impedance, which is:

$$Z_p = Z_{Base}\cdot Z_{pu}$$
$$= (480\Omega)(1.350pu) = 648\Omega$$

Calculate the load resistance from actual voltage and current:

$$\hat{Z}_p = \frac{62.4kV}{96.2A} = 648\Omega$$

$$\hat{Z}_p = Z_p = 648\Omega$$

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4.2 Changing the Base of Per-Unit Quantities

Often the per-unit impedance of a component of a system is expressed on a base other than the one selected as base for the part of the system in which the component is located. Since all impedances in any one part of a system must be expressed on the same impedance base when making computations, it is necessary to have a means of converting per-unit impedances from one base to another. From Equations (5) and (7) the per unit impedance can be given as:

$$Z_{pu} = \frac{Z_{p-Actual}}{Z_{Base}} = \frac{(Z_{p-Actual})(S_{3\phi-Base})}{(V_{LL-Base})^2} \tag{9}$$

Equation (9) shows that per-unit impedance is directly proportional to base power and inversely proportional to the square of the base voltage. Therefore, to change from per-unit impedance on a given base to per-unit impedance on a new base, the following equation applies:

$$Z_{new-pu} = Z_{old-pu}\left(\frac{S_{3\phi-Base-new}}{S_{3\phi-Base-old}}\right)\left(\frac{V_{LL-Base-old}}{V_{LL-Base-new}}\right)^2 \tag{10}$$

Equation (10) is important in changing the per-unit impedance given on a particular base to a new base. In some problems, when transformers are involved we must choose more than one voltage base for each primary and secondary side of the transformers and one power base for the entire system. The following examples will illustrate the procedure.

EXAMPLE 4.3

A three-phase 13.0 kV transmission line delivers 8 MVA at 13.6 kV to a resistive load. The per phase impedance of the line is (0.01 + j0.05) p.u. on a 13.0 kV, 8 MVA base. What is the voltage drop across the line in per unit and in volts?

a) Choose the base voltage to be 13 kV and the base power equal to 8 MVA.

The base current and the load current are:

$$I_{Base} = \frac{8.0MVA}{\sqrt{3}\cdot 13.0kV} = 355.3A$$

$$I_{Load} = \frac{8.0MW}{\sqrt{3}\cdot 13.6kV} = 339.6A\angle 0^\circ$$

$$I_{Load-pu} = \frac{339.6A\angle 0^\circ}{355.3A} = 0.956\,pu\angle 0^\circ$$

The voltage drop is calculated as:

$$\begin{aligned} V_{drop} &= I_{line}\cdot Z_{line} \\ &= (0.956\angle 0^\circ)(0.01 + j0.05) = 0.00956 + j0.0478 = 0.0487\angle 78.7^\circ \end{aligned}$$

b) The actual values for the line-to-line voltage drop and the phase voltage drop are:

$$V_{drop-LL} = (0.0487\angle 78.7^\circ)(13.0kV) = 633V\angle 78.7^\circ$$

$$V_{drop-p} = (0.0487\angle 78.7^\circ)\frac{(13.0kV)}{\sqrt{3}} = 366V\angle 78.7^\circ$$

EXAMPLE 4.4

The per phase reactance of a three-phase, 220 kV, 6.25 kVA transmission line is 8.4ohm. Find the reactance value in per unit, based on the rated values of the line. Convert the per unit reactance value to a 230 kV, 7.5 kVA base.

a)

$$Z_{Base} = \frac{(V_{LL-Base})^2}{S_{3\phi-Base}}$$

$$= \frac{(220{,}000V)^2}{6{,}250{,}000VA} = 7.744\Omega$$

$$X_{pu} = \frac{X}{Z_{Base}}$$

$$= \frac{8.4\Omega}{7.744\Omega} = 1.085\,pu$$

b)

$$X_{new-pu} = X_{old-pu}\left(\frac{S_{Base-new}}{S_{Base-old}}\right)\left(\frac{V_{Base-old}}{V_{Base-new}}\right)^2$$

$$= 1.085\,pu\left(\frac{7.5kVA}{6.25kVA}\right)\left(\frac{220kV}{230kV}\right)^2 = 1.19\,pu$$

EXAMPLE 4.5

Consider the system in Figure 1 below. Find the new per-unit values for each element of the system based on a 2.0 MVA system base. Draw the impedance diagrams of the system.

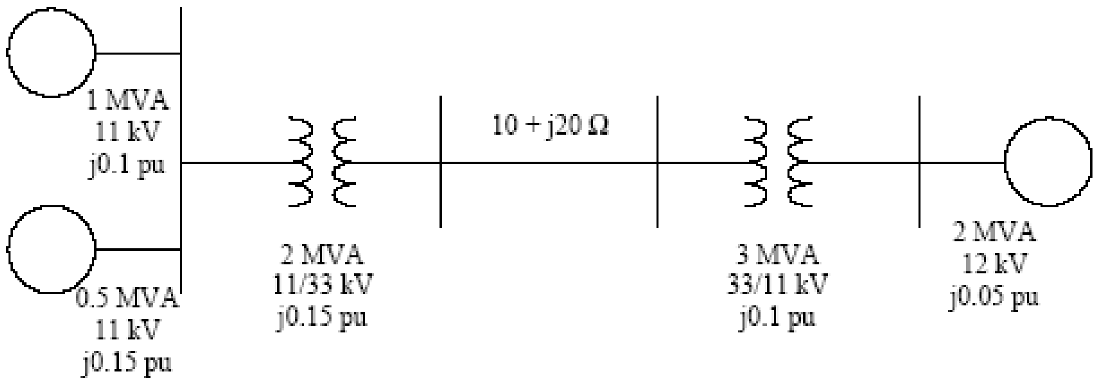

Figure 1 Small Power System of Example 3.5.

The per-unit values for each element of the three-phase system shown above are as follows:

Machine 1	1.00 MVA, 11 kV, Z = j0.1 pu
Machine 2	0.50 MVA, 11 kV, Z = j0.15 pu
Machine 3	2.00 MVA, 12 kV, Z = j0.05 pu
Transmission Line	Z = 10 + j20 ohm
Transformer 1	2.00 MVA, 11 / 33 kV, Z = j0.15 pu
Transformer 2	3.00 MVA, 33 / 11 kV, Z = j0.10 pu

a) The power base for the entire system is 2.00 MVA. The base voltages are chosen for the following areas as:

Zone 1 $V_{B1} = 11kV$

Zone 2 $V_{B2} = \dfrac{11}{11/33} = 33kV$

Zone 3 $V_{B3} = \dfrac{33}{33/11} = 11kV$

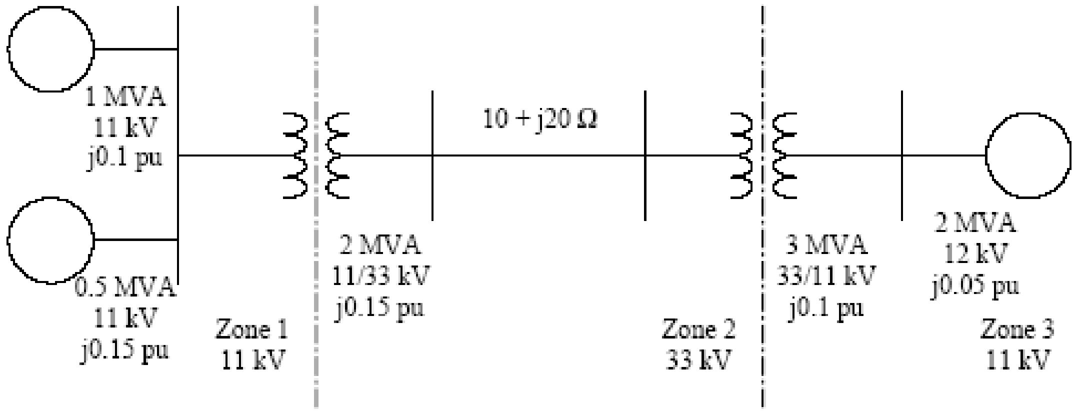

Figure 2 Three Voltage Zones of Example 3.5.

The per-unit values for each element of the above system can be obtained by using Equation 10 as follow:

Machine 1: $$Z_{pu-new} = j0.10\frac{2.0MW}{1.0MW}\left(\frac{11kV}{11kV}\right)^2 = j0.20\,pu$$

Machine 2: $$Z_{pu-new} = j0.15\frac{2.0MW}{0.5MW}\left(\frac{11kV}{11kV}\right)^2 = j0.60\,pu$$

Transformer 1: $$Z_{pu-new} = j0.15\,pu$$

Transmission Line: $$Z_{Base} = \frac{(33kV)^2}{2.0MVA} = 544.5\Omega$$

$$Z_{pu} = \frac{j20\Omega}{544.5\Omega} = j0.037\,pu$$

Transformer 2: $$Z_{pu-new} = j0.10\frac{2.0MW}{3.0MW}\left(\frac{33kV}{33kV}\right)^2 = j0.067\,pu$$

Machine 3: $$Z_{pu-new} = j0.05\frac{2.0MW}{2.0MW}\left(\frac{12kV}{11kV}\right)^2 = j0.06\,pu$$

b) The reactance diagram is shown in Figure 3.

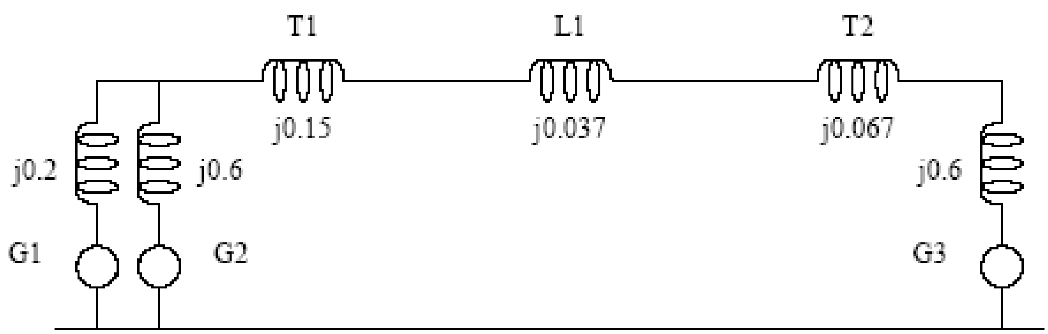

Figure 3 Reactance Diagram of Example 3.5.

<u>EXAMPLE 4.6</u>

Draw an impedance diagram for the electric power system shown in Figure 4 showing all impedances in per unit on a 100-MVA base. Choose 20 kV as the voltage base for generator. The three-phase power and line-line ratings are given below.

***G*1 :**	**90 MVA**	**20 kV**	***X* = 9%**
***T*1 :**	**80 MVA**	**20/200 kV**	***X* = 16%**
***T*2 :**	**80 MVA**	**200/20 kV**	***X* = 20%**
***G*2 :**	**90 MVA**	**18 kV**	***X* = 9%**
Line:		**200 kV**	***X* = 120 ohm**
Load:		**200 kV**	***S* = 48 MW +*j*64 Mvar**

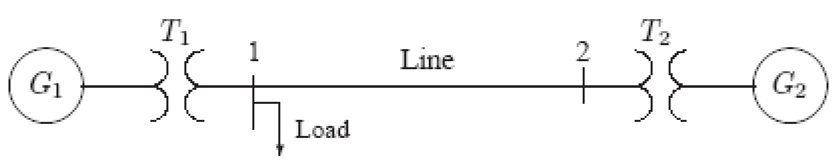

Figure 4 One-line diagram

Solution:

The base voltage V_{BG1} on the LV side of $T1$ is 20 kV. Hence the base on its HV side is

$$V_{B1} = 20\left(\frac{200}{20}\right) = 200 \quad \text{kV}$$

This fixes the base on the HV side of $T2$ at V_{B2} = 200 kV, and on its LV side at

$$V_{BG2} = 200\left(\frac{20}{200}\right) = 20 \quad \text{kV}$$

The generator and transformer reactances in per unit on a 100 MVA base, are:

$$G\colon \qquad X = 0.09\left(\frac{100}{90}\right) = 0.10 \ \text{pu}$$

$$T_1\colon \qquad X = 0.16\left(\frac{100}{80}\right) = 0.20 \ \text{pu}$$

$$T_2\colon \qquad X = 0.20\left(\frac{100}{80}\right) = 0.25 \ \text{pu}$$

$$G_2\colon \qquad X = 0.09\left(\frac{100}{90}\right)\left(\frac{18}{20}\right)^2 = 0.081 \ \text{pu}$$

The base impedance for the transmission line is

$$Z_{BL} = \frac{(200)^2}{100} = 400 \ \ \Omega$$

The per unit line reactance is

$$Line\colon \qquad X = \left(\frac{120}{400}\right) = 0.30 \ \text{pu}$$

The load impedance in ohms is

$$Z_L = \frac{(V_{L-L})^2}{S^*_{L(3\phi)}} = \frac{(200)^2}{48 - j64} = 300 + j400 \ \ \Omega$$

The load impedance in per unit is

$$Z_{L(pu)} = \frac{300 + j400}{400} = 0.75 + j1.0 \ \text{pu}$$

The per unit equivalent circuit is shown in Figure 5.

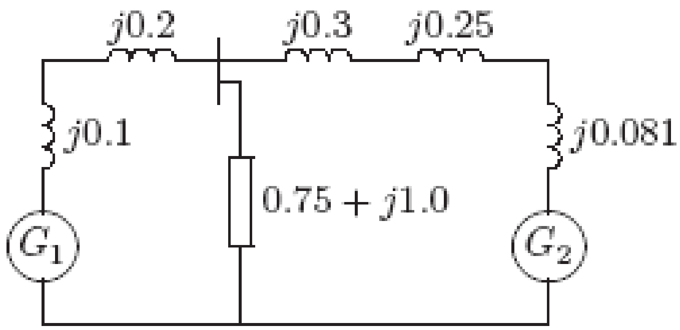

Figure 5 Per unit impedance diagram

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<u>EXAMPLE 4.7</u>

The one-line diagram of a three-phase power system is as shown in Figure 6. Impedances are marked in per unit on a 100MVA, 400kV base. The load at bus 2 is $S2$ = 15.93 MW-j33.4 Mvar, and at bus 3 is $S3$ = 77 MW +j14 Mvar. It is required to hold the voltage at bus 3 at $400\angle 0^\circ$ kV. Working in per unit, determine the voltage at buses 2 and 1.

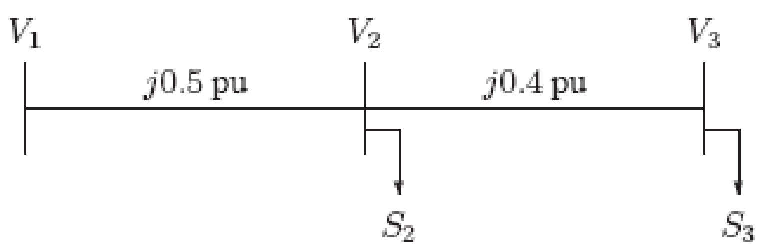

Figure 6 One-line diagram

Solution:

$$
\begin{aligned}
S_2 &= 15.93\ \text{MW} - j33.4\ \text{Mvar} = 0.1593 - j0.334\ \ \text{pu} \\
S_3 &= 77.00\ \text{MW} + j14.0\ \text{Mvar} = 0.7700 + j0.140\ \ \text{pu} \\
V_3 &= \frac{400\angle 0^\circ}{400} = 1.0\angle 0^\circ\ \ \text{pu} \\
I_3 &= \frac{S_3^*}{V_3^*} = \frac{0.77 - j0.14}{1.0\angle 0^\circ} = 0.77 - j0.14\ \ \text{pu} \\
V_2 &= 1.0\angle 0^\circ + (j0.4)(0.77 - j0.14) = 1.1\angle 16.26^\circ\ \ \text{pu}
\end{aligned}
$$

Therefore, the line-to-line voltage at bus 2 is

$$
\begin{aligned}
V_2 &= (400)(1.1) = 440\ \ \text{kV} \\
I_2 &= \frac{S_2^*}{V_2^*} = \frac{0.1593 + j0.334}{1.1\angle -16.26^\circ} = 0.054 + j0.332\ \ \text{pu} \\
I_{12} &= (0.77 - j0.14) + (0.054 + j0.332) = 0.824 + j0.192\ \ \text{pu} \\
V_1 &= 1.1\angle 16.26^\circ + (j0.5)(0.824 + j0.192) = 1.2\angle 36.87^\circ\ \ \text{pu}
\end{aligned}
$$

Therefore, the line-to-line voltage at bus 1 is

$$V_1 = (400)(1.2) = 480\ \ \text{kV}$$

EXAMPLE 4.8

The one-line diagram of a three-phase power system is as shown in Figure 7. The transformer reactance is 20 percent on a base of 100-MVA, 23/115-kV and the line impedance is $Z = j66.125$ohm. The load at bus 2 is $S2$=184.8 MW $+j6.6$ Mvar, and at bus 3 is $S3$ = 0MW+$j20$ Mvar. It is required to hold the voltage at bus 3 at $115\angle 0^\circ$ kV. Working in per unit, determine the voltage at buses 2 and 1.

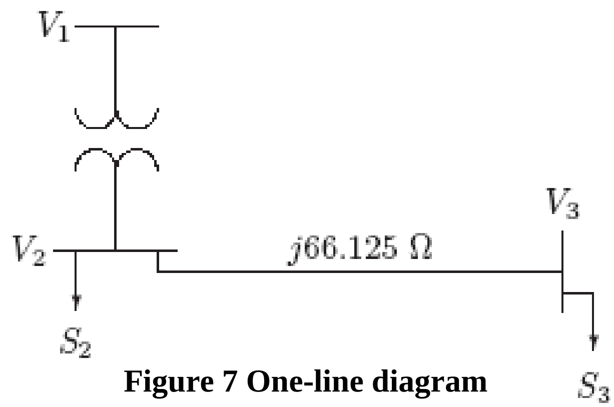

Figure 7 One-line diagram

<u>Solution:</u>

$$S_2 = 184.8\text{ MW} + j6.6\text{ Mvar} = 1.848 + j0.066\ \text{ pu}$$
$$S_3 = \quad 0\text{ MW} + j20.0\text{ Mvar} = \quad 0 + j0.20\ \text{ pu}$$

$$V_3 = \frac{115\angle 0^\circ}{115} = 1.0\angle 0^\circ\ \text{ pu}$$
$$I_3 = \frac{S_3^*}{V_3^*} = \frac{-j0.2}{1.0\angle 0^\circ} = -j0.2\ \text{ pu}$$
$$V_2 = 1.0\angle 0^\circ + (j0.5)(-j0.2) = 1.1\angle 0^\circ\ \text{ pu}$$

Therefore, the line-to-line voltage at bus 2 is

$$V_2 = (115)(1.1) = 126.5\ \text{ kV}$$

$$I_2 = \frac{S_2^*}{V_2^*} = \frac{1.848 - j0.066}{1.1\angle 0^\circ} = 1.68 - j0.06\ \text{ pu}$$
$$I_{12} = (1.68 - j0.06) + (-j0.2) = 1.68 - j0.26\ \text{ pu}$$
$$V_1 = 1.1\angle 0^\circ + (j0.2)(1.68 - j0.26) = 1.2\angle 16.26^\circ\ \text{ pu}$$

Therefore, the line-to-line voltage at bus 1 is

$$V_1 = (23)(1.2) = 27.6\ \text{ kV}$$