* Sequential Switching *

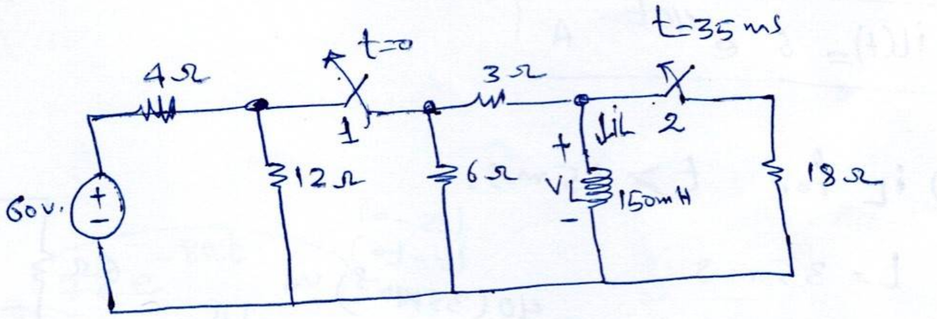
مفاتيح متتالية

Ex: The two switches in the cct. have been closed along time. At $t=0$, switch 1 is opend. Then, 35 ms later, switch 2 is opend.

a) Find $i_L(t)$ for $0 \leq t \leq 35\,ms$.

b) Find i_L for $\geq 35\,ms$.

c) what percentage of the initial energy storde in the 150 mH L is dissipated in the 18 Ω R.?



Sol

$t=0 \longrightarrow S_1$ opened, after 35 ms $\longrightarrow S_2$ is opened

that's mean at $t<0$, all the switches were closed for along time

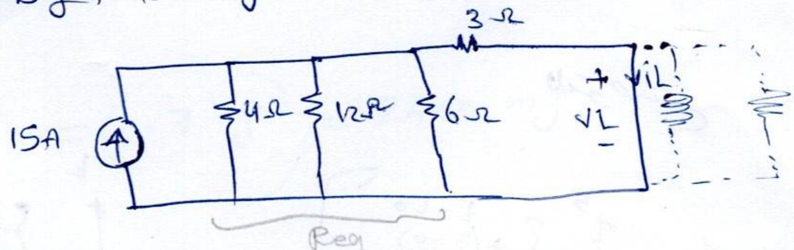
S_1 & S_2 → close

So $\longrightarrow$ at $t<0$

$I_0 \longrightarrow$ we can convert the voltage sourse to current source.

$$I_s = \frac{V_s}{R} = \frac{60}{4} = 15\,A$$

∴ the cct will become by this fig



∴ $I_0 = I_{3\Omega}$

$R_{eq} = 6 // 12 // 4 = 2\,\Omega$

$$I_0 = I_{3\Omega} = 15 * \frac{2}{3+2} \quad \Rightarrow \quad I_0 = I_s * \frac{R_{eq}}{R_{eq}+3}$$

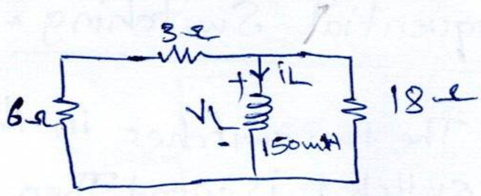
$$I_0 = 6\,A$$

a) $i_L(t)$ for $0 \leq t \leq 35\,ms$

$S_1 \longrightarrow$ open

$S_2 \longrightarrow$ close

$\Longrightarrow$



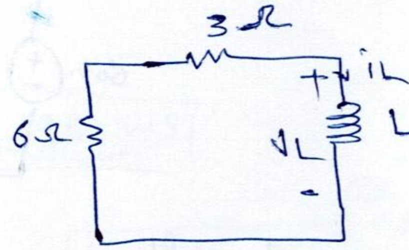
$R_T = (3+6) \parallel 18 = 6\,\Omega$

$\tau = \frac{L}{R} = \frac{150 \times 10^{-3}}{6} = 25\,ms \quad , \quad \frac{1}{\tau} = 40\,s$

$i_L(t) = I_o\, e^{-t/\tau}$

$\boxed{i_L(t) = 6\, e^{-40t} \quad A}$

b) i_L for $t \geq 35\,ms$.



$t = 35\,ms$ $\quad (35-0)$ $\quad (t - t_o)$

$i_L(35ms) = 6\, e^{-40(35 \times 10^{-3})} = 6e^{-1.4}$

$= 1.48 \quad A$ initial ← الفترة $t \geq 35 \quad t)$

$t \geq 35 \times 10^{-3}$

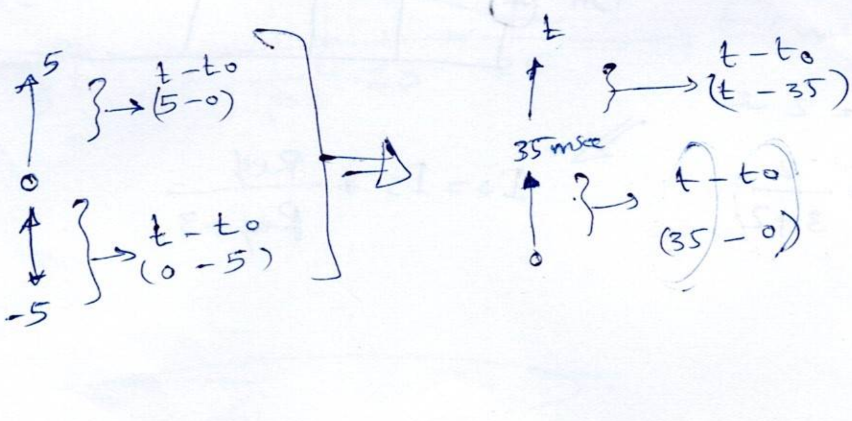
$R_T = 3 + 6 = 9\,\Omega$

$\tau = \frac{L}{R} = \frac{150 \cdot 10^{-3}}{9} = 16.67\,ms \Rightarrow \frac{1}{\tau} = 60\,s$

$\boxed{\therefore i_L(t) = 1.48\, e^{-60(t - 0.035)} \quad A} \quad , t \geq 35\,ms$

$\rightarrow i(t) = I_o e^{-(t-t_o)/\tau}$

للتوضيح



تصبح initial
للقيمة الجديدة

~~e~~ ⓒ $\frac{w_{18\Omega}}{w_0(L)}$

The R of 18Ω in cct only during the interval $0 \leq t \leq 35\,ms$

So, The current was $\boxed{i_L(t) = 6e^{-40t}\ A}$

$$V_L = L\frac{di(t)}{dt} \Rightarrow V_L = (150 * 10^{-3})\frac{d(6e^{-40t})}{dt}$$

$$V_L = (150.10^{-3})(-240\, e^{-40t})$$

$$V_L = -36\, e^{-40t}\ V.$$

$$P_{18\Omega} = \frac{V_L^2}{18\Omega} = \frac{V^2}{R}$$

$$= \frac{(-36\, e^{-40t})^2}{18} = 72\, e^{-80t}\ W$$

$$W_{18\Omega} = \int_0^{0.035} p\, dt$$

$$= \int_0^{0.035} (72\, e^{-80t})\, dt = -\frac{72}{80}\, e^{-80t}\Big|_0^{0.035} = 0.9(1-e^{-2.8})$$

$$\boxed{W_{18\Omega} = 845.27\ mJ}$$

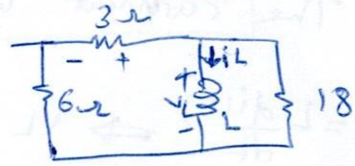
$$w_0 = \frac{1}{2} L\, I_0^2$$

$$= \frac{1}{2}.(150.10^{-3}).(6)^2 = 2.7\ J = 2700\ mJ$$

$$\therefore \frac{W_{18\Omega}}{W_0(t)} = \frac{845.27 * 10^{-3}}{2.7} * 100\,\% = 31.31\,\%$$

d) Repeat (c) for the 3Ω resistor.

For $\boxed{0 < t < 35\ msec}$, the voltage across the 3Ω resistor is



$$V_{3\Omega} = V_L * \frac{3}{3+6}$$

$$= \cancel{36}^{12} e^{-40t} * \frac{\cancel{3}^{1}}{\cancel{9}_{3}} = -12 e^{-40t}\ V.$$

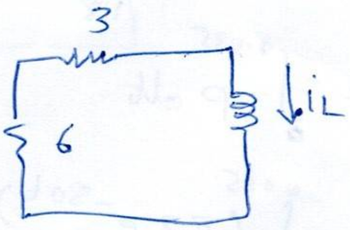
$$P_{3\Omega} = \frac{V^2}{R} = \frac{(-12 e^{-40t})^2}{3} = \frac{144 e^{-80t}}{3}\ w$$

$$w_{3\Omega} = \int_0^{0.035} \left(\frac{144 e^{-80t}}{3}\right) dt$$

$$= \frac{-144}{3 \times 80} \left[e^{-80t}\right]_0^{0.035}$$

$$= 0.6(1 - e^{-2.8}) = 563.51\ mJ$$

For $\boxed{t > 35\ ms}$.



$$i_L = i_{3\Omega} = (1.48 e^{-60(t-0.035)})\ A$$

$$P_{3\Omega} = i^2 * R$$
$$= (1.48 e^{-60(t-0.035)})^2 * 3$$

$$W_{3\Omega} = \int_{0.035}^{\infty} (1.48 e^{-60(t-0.035)})^2 * 3\ dt$$

$$w_{3\Omega} = 54.73\ mJ$$

* The total energy dissipated in the 3Ω resistor is

$$w_{3\Omega}(total) = 563.51 + 54.73 = \boxed{618.24\ mJ}$$

$$\therefore \frac{w_{3\Omega}}{w_L} * \% = \frac{618.24}{2700} * 100\% = 22.90\ \%$$

(e) Repeat for 6Ω

∵ المقاومة 6Ω هي مربوطة على التوالي مع 3Ω ، إذن الطاقة المصروفة هي ضعف الموجودة في 3Ω

$$w_{6\Omega}(total) = 2 * 618.24 = 1236.48\ mJ$$

45.80% = 2 * 22.90 ⇐ والنسبة أيضا ضعف

* للتحقق من صحة الحسابات

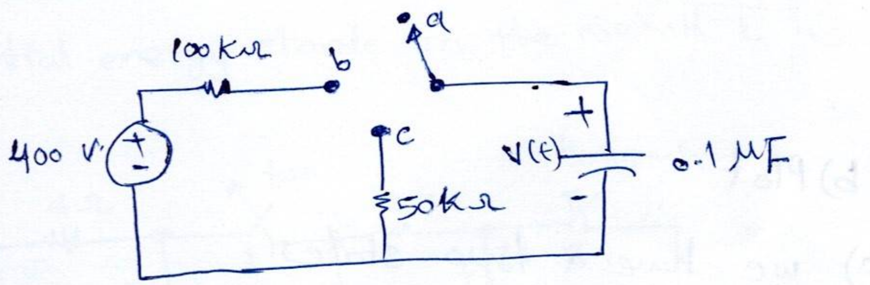
$$\underset{6\Omega}{1236.48} + \underset{3\Omega}{618.24} + \underset{18\Omega}{845.27} = 2699.99\ mJ \simeq 2700\ mJ$$

and $\underset{18\Omega}{31.31} + \underset{3\Omega}{22.90} + \underset{6}{45.80} = 100.01\ \%$

The uncharge capacitor in the cct. is initially switched to the terminal (a) of the 3 position switch

Ex: At $t=0$, the switch is moved to position b, where it remains for 15 ms. after the 15 ms delay, the switch is moved to c, where it remains indefinitely.

a) Derive $V_c(t)$ b) Plot V_c & t c) t at $V_c = 200$ v.



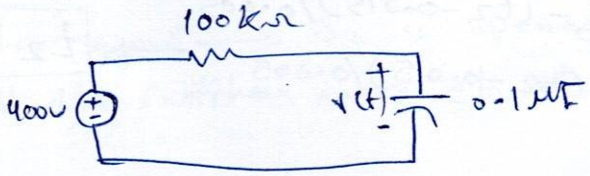
Sol:-

at $t<0$

$V_0 = 0$ ⇒ لأن السعة غير مشحونة بالبداية (الدائرة غير موصلة)

at $t \geq 0$ →

(0 ⟶ 15 ms)



(step)

$\boxed{at\ 0 \leq t \leq 15\,m}$

$V_c = V_s = 400$ v.

$\tau = RC = 100 \cdot 10^3 * 0.1 * 10^{-6} = 10\,ms$, $\frac{1}{\tau} = 100\,s$

$\boxed{\text{step ckt}}$

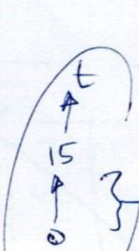
$V_0 = 0$

$V_f = 400$

$x(t) = x_f + (x_0 - x_f) e^{-t/\tau}$

$\boxed{v(t) = V_f + (V_0 - V_f) e^{-t/\tau}}$

$\boxed{V(t) = 400 + (0 - 400) e^{-100t}}$



$t - t_0$
$(15 - 0)$

at $0 \leq t \leq 15$:

$v(t) = 400 - 400 e^{-100(t-t_0)}$

$= 400 - 400 e^{-100(15*10^{-3} - 0)}$ ⇒ $= 400 - 400 (0.223)$

$\boxed{V(t) = 310.75\ v.}$

at t in position c

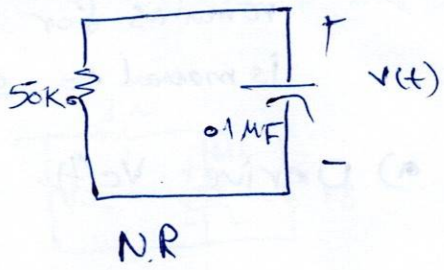
$t \geq 15$

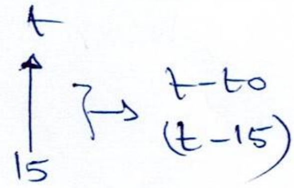
$\tau = RC = 50 * 10^3 * 0.1 * 10^{-6} = 5\,ms$

$v(t) = V_o e^{-t/\tau}$

$$\boxed{v(t) = 310.75\, e^{-(t-0.015)/0.005}\ v. \quad t \geq 15\,ms}$$

$t > 15$





b) Plot

c) we have a two states

$v(t) = 400 - 400\, e^{-100 t_1}$

$200 = 400 - 400\, e^{-100 t_1} \Longrightarrow \boxed{t_1 = 6.93\ ms}$

$v(t) = 310.75\, e^{-(t_2 - 0.015)/0.005} \Longrightarrow \boxed{t_2 = 17.20\ ms}$

$200 = 310.75\, e^{-(t_2 - 0.015)/0.005}$